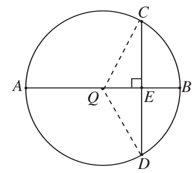
Unit VI — Circles

Part C — Line and Segment Relationships

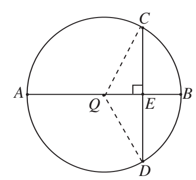
p. 555 – Lesson 1 — Theorem 73



1. Theorem 73 - "If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects that chord."

Given: $\overline{AB}$ is a diameter of $\odot Q$; $\overline{AB} \perp \overline{CD}$
Prove: $\overline{AB}$ bisects $\overline{CD}$

STATEMENT	REASONS
1. $\overline{AB}$ is a diameter of $\odot Q$	1. Given
2. $\overline{AB} \perp \overline{CD}$	2. Given
3. Draw radius $\overline{CQ}$	3. Postulate 2 - For any two different points, there is exactly one line containing them
4. Draw radius $\overline{DQ}$	4. Postulate 2
5. $\overline{DQ} \cong \overline{CQ}$	5. Radii of the same circle are congruent
6. $\angle QEC$ is a right angle	6. Definition of Perpendicular Lines
7. $\triangle QEC$ is a right triangle	7. Definition of Right Triangle
8. $\angle QED$ is a right angle	8. Definition of Perpendicular Lines
9. $\triangle QED$ is a right triangle	9. Definition of Right Triangle
10. $\overline{QE} \cong \overline{QE}$	10. Reflexive Property for Congruent Segments
11. $\triangle QEC \cong \triangle QED$	11. Postulate Corollary 13c - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two right triangles are congruent. (HL)
12. $\overline{CE} \cong \overline{DE}$	12. C.P.C.T.C.
13. $\overline{AB}$ bisects $\overline{CD}$	13. Definition of Segment Bisector



2. Corollary 73a - "If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects the arcs intercepted by that chord."

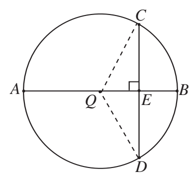
Given: $\overline{AB}$ is a diameter of $\odot Q$; $\overline{AB} \perp \overline{CD}$
Prove: $\overline{AB}$ bisects $\overset{\frown}{CD}$; $\overline{AB}$ bisects $\overset{\frown}{CAD}$

STATEMENT	REASONS
1. $\overline{AB}$ is a diameter of $\odot Q$	1. Given
2. $\overline{AB} \perp \overline{CD}$	2. Given
3. Draw radius $\overline{CQ}$	3. Postulate 2 - For any two different points, there is exactly one line containing them
4. Draw radius $\overline{DQ}$	4. Postulate 2
5. $\overline{DQ} \cong \overline{CQ}$	5. Radii of the same circle are congruent
6. $\angle QEC$ is a right angle	6. Definition of Perpendicular Lines
7. $\triangle QEC$ is a right triangle	7. Definition of Right Triangle
8. $\angle QED$ is a right angle	8. Definition of Perpendicular Lines
9. $\triangle QED$ is a right triangle	9. Definition of Right Triangle
10. $\overline{QE} \cong \overline{QE}$	10. Reflexive Property for Congruent Segments
11. $\triangle QEC \cong \triangle QED$	11. Postulate Corollary 13c - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two right triangles are congruent. (HL)
12. $\angle CQB \cong \angle DQB$	12. C.P.C.T.C.
13. $\overset{\frown}{CB} \cong \overset{\frown}{DB}$	13. Theorem 65 - If, in the same circle, or in congruent circles, two central angles are congruent, then their intercepted minor arcs are congruent.
14. $\overline{AB}$ bisects $\overset{\frown}{CD}$	14. Definition of Arc Bisector
15. $m\overset{\frown}{AC} + m\overset{\frown}{CB} = m\overset{\frown}{ACB}$	15. Postulate 8 - Arc Addition Postulate - Fourth Assumption
16. $m\overset{\frown}{AD} + m\overset{\frown}{DB} = m\overset{\frown}{ADB}$	16. Postulate 8
17. $\overset{\frown}{ACB}$ is a semicircle	17. Definition of Semicircle - An arc of a circle is a semicircle, if and only if, it is an arc whose endpoints are the endpoints of a diameter of the circle.
18. $m\overset{\frown}{ACB} = 180°$	18. Definition of Semicircle - an arc which is one-half of a circle. (One-half of a complete rotation)

2. continued

STATEMENT	REASONS
19. $\overset{\frown}{ADB}$ is a semicircle	19. Definition of Semicircle
20. $m\overset{\frown}{ADB} = 180^\circ$	20. Definition of Semicircle
21. $m\overset{\frown}{ACB} = m\overset{\frown}{ADB}$	21. Substitution
22. $m\overset{\frown}{AC} + m\overset{\frown}{CB} = m\overset{\frown}{AD} + m\overset{\frown}{DB}$	22. Substitution
23. $m\overset{\frown}{CB} = m\overset{\frown}{DB}$	23. Definition of Congruent Arcs
24. $m\overset{\frown}{AC} = m\overset{\frown}{AD}$	24. Addition (Subtraction) Property of Equality
25. $\overset{\frown}{AC} \cong \overset{\frown}{AD}$	25. Definition of Congruent Arcs
26. $\overline{AB}$ bisects $\overset{\frown}{CAD}$	26. Definition of Arc Bisector

3. "If the midpoints of the two arcs of a circle determined by a chord are joined by a line segment, then the line segment is the perpendicular besector of the chord."



Given: $\overset{\frown}{CD}$ and $\overset{\frown}{CAD}$ are two arcs determined by chord $\overline{CD}$.
Point A and point B are midpoints of $\overset{\frown}{CD}$ and $\overset{\frown}{CAD}$ joined by $\overline{AB}$.
Prove: $\overline{AB} \perp \overline{CD}$; $\overline{AB}$ bisects $\overline{CD}$

STATEMENT	REASONS
1. $\overset{\frown}{CD}$ and $\overset{\frown}{CAD}$ are two arcs determined by chord $\overline{CD}$.	1. Given
2. Point A and point B are midpoints of $\overset{\frown}{CD}$ and $\overset{\frown}{CAD}$ joined by $\overline{AB}$.	2. Given
3. $\overset{\frown}{CB} \cong \overset{\frown}{DB}$	3. Definition of midpoint
4. $\overset{\frown}{AC} \cong \overset{\frown}{AD}$	4. Definition of midpoint
5. $m\overset{\frown}{CB} = m\overset{\frown}{DB}$	5. Definition of Congruent Arcs
6. $m\overset{\frown}{AC} = m\overset{\frown}{AD}$	6. Definition of Congruent Arcs
7. $m\overset{\frown}{AC} + m\overset{\frown}{CB} + m\overset{\frown}{BD} + m\overset{\frown}{DA} = 360$	7. Postulate 8 - First Assumption - The set of all points on a circle can be put into a one-to-one correspondence with the real numbers from 0 to 360, inclusive, with the exception of any one point which may be paired with 0 and 360.
8. $m\overset{\frown}{AC} + m\overset{\frown}{CB} + m\overset{\frown}{CB} + m\overset{\frown}{AC} = 360$	8. Substitution
9. $2m\overset{\frown}{AC} + 2m\overset{\frown}{CB} = 360$	9. Properties of Algebra - Collect like Terms
10. $m\overset{\frown}{AC} + m\overset{\frown}{CB} = 180$	10. Multiplication Property of Equality
11. $m\overset{\frown}{AC} + m\overset{\frown}{CB} = m\overset{\frown}{ACB}$	11. Postulate 8 - Fourth Assumption - Arc Addition Assumption
12. $m\overset{\frown}{ACB} = 180$	12. Substitution
13. $\overset{\frown}{ACB}$ is a semicircle	13. Definition of Semicircle - "...a semicircle is the intercepted arc of a central angle of 180."
14. $\overline{AB}$ is a diameter	14. Definition of Semicircle - An arc of a circle is a semicircle, if and only if, it is an arc whose endpoints are the endpoints of a diameter of a circle.
15. $\overline{AB}$ passes through point Q, the center of the circle	15. Definition of diameter
16. Draw radius $\overline{QC}$	16. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
17. Draw $\overline{QD}$	17. Postulate 2
18. $\overline{QC} \cong \overline{QD}$	18. Radii of the same circle are congruent
19. $m\angle CQB = m\overset{\frown}{CB}$	19. Definition of Measure of a Central Angle - the measure of a minor arc and the measure of a central angle are equal.
20. $m\angle DQB = m\overset{\frown}{DB}$	20. Definition of Measure of a Central Angle
21. $m\angle CQB = m\angle DQB$	21. Substitution
22. $\angle CQB \cong \angle DQB$	22. Definition of Congruent Angles
23. $\overline{QE} \cong \overline{QE}$	23. Reflexive Property of Congruent Segments
24. $\triangle CQE \cong \triangle DQE$	24. Postulate 13 - Triangle Congruence - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent. (SAS Congruence Postulate)
25. $\angle QEC \cong \angle QED$	25. C.P.C.T.C.
26. $\angle QEC$ and $\angle QED$ are supplementary angles	26. Theorem 10 - If the exterior sides of two adjacent angles are opposite rays, then the two angles are supplementary.
27. $\angle QEC$ and $\angle QED$ are right angles	27. Corollary 10b - If two angles are supplementary and congruent, then each angle is a right angle.
28. $\overline{AB} \perp \overline{CD}$	28. Definition of Perpendicular Lines (Segments)
29. $\overline{CE} \cong \overline{DE}$	29. C.P.C.T.C.
30. Point E is the midpoint of $\overline{CD}$	30. Definition of Midpoint of a Line Segment
31. $\overline{AB}$ is the bisector of $\overline{CD}$	31. Definition of Bisector of a Line Segment. Q.E.D.

4. A line segment is the perpendicular bisector of chord of a circle if and only if the line segment joins the midpoints of the two arcs determined by the chord.
5. $\overline{PR}$
6. Point P
7. Point Q
8. $\overset{\frown}{RX}$
9. $\overset{\frown}{TY}$
10. $\overline{XY}$
11. $\overline{QU}$
12. $\overline{XY}$
13. No
14. x = 9 Theorem 73
15. x = 34° Corollary 73a

16.
$$(BX)^2 = (BC)^2 + (CX)^2$$
$$(7)^2 = (4)^2 + (CX)^2$$
$$49 = 16 + (CX)^2$$
$$33 = (CX)^2$$
$$\pm\sqrt{33} = CX$$
(CX cannot be negative)
$$\sqrt{33} = CX$$

$$CX + CY = XY$$
$$CX = CY$$
$$\sqrt{33} + \sqrt{33} = XY$$
$$2\sqrt{33} = XY$$

17. Draw radius $\overline{QD}$
QD = QA
QD = 13

$$DM = \frac{1}{2} \cdot DC$$
$$DM = \frac{1}{2} \cdot 24$$
$$DM = 12$$

$$(QD)^2 = (DM)^2 + (QM)^2$$
$$(13)^2 = (12)^2 + (QM)^2$$
$$169 = 144 + (QM)^2$$
$$25 = (QM)^2$$
$\pm\sqrt{25} = QM$ (QM cannot be negative)
$$5 = QM$$

18. Draw radius $\overline{QY}$
$$CY = \frac{1}{2} \cdot XY$$
$$CY = \frac{1}{2} \cdot 18$$
$$CY = 9$$

$$(QY)^2 = (QC)^2 + (CY)^2$$
$$(QY)^2 = (9)^2 + (9)^2$$
$$(QY)^2 = 81 + 81$$
$$(QY)^2 = 162$$
$QY = \pm\sqrt{162}$ (QY cannot be negative)
$$QY = \sqrt{81} \cdot \sqrt{2}$$
$$QY = 9\sqrt{2}$$

19.
$$m\overset{\frown}{UW} = 360 - 288$$
$$= 72$$

$$m\angle VQW = m\overset{\frown}{VW}$$
$$m\angle VQW = 36$$

$$m\overset{\frown}{VW} = \frac{1}{2} \cdot m\overset{\frown}{UW}$$
$$= \frac{1}{2} \cdot 72$$
$$= \frac{1 \cdot \cancel{2} \cdot 36}{\cancel{2}}$$
$$= 36$$

20. BX = 7 Theorem 73
DY = 8 Theorem 73
$m\overset{\frown}{AD} = 64$

$$m\overset{\frown}{BE} = 58$$
$$m\overset{\frown}{BE} = m\overset{\frown}{AE}$$
$$m\overset{\frown}{AD} = 180 - \left(m\overset{\frown}{BE} + m\overset{\frown}{AE}\right)$$
$$= 180 - (58 + 58)$$
$$= 180 - 116$$
$$= 64$$

21. $m\overset{\frown}{AB} = 80$
CD = $10\sqrt{5}$

$$m\overset{\frown}{BH} = 180 - m\overset{\frown}{GB}$$
$$= 180 - 140$$
$$= 40$$

$m\overset{\frown}{BH} = m\overset{\frown}{AH}$ Corollary 73a
$$m\overset{\frown}{AB} = m\overset{\frown}{BH} + m\overset{\frown}{AH}$$
$$m\overset{\frown}{AB} = 40 + 40$$
$$m\overset{\frown}{AB} = 80$$

If AB = 18, then HB = 9 (Theorem 73)
QI = 12 (Given)
Draw $\overline{QB}$

$$(QB)^2 = (HB)^2 + (QI)^2$$
$$(QB)^2 = (9)^2 + (12)^2$$
$$(QB)^2 = 81 + 144$$
$$(QB)^2 = 225$$
$$QB = 15$$

$$(QC)^2 = (JC)^2 + (QJ)^2$$
$$(15)^2 = (JC)^2 + (10)^2$$
$$225 = (JC)^2 + 100$$
$$125 = (JC)^2$$
$\pm\sqrt{125} = JC$ (JC cannot be negative)
$$\sqrt{25} \cdot \sqrt{5} = JC$$
$$5\sqrt{5} = JC$$

So, QC = 15

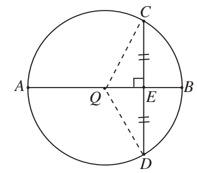
$$CD = 2 \cdot JC = 2 \cdot 5\sqrt{5} = 10\sqrt{5}$$

Unit VI — Circles

Part C — Line and Segment Relationships

p. 559 – Lesson 2 — Theorem 74 & 75

1. Theorem 74 - "If a diameter of a circle bisects a chord that is not a diameter, then that diameter is perpendicular to that chord."

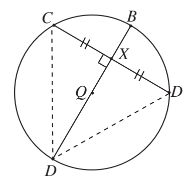


Given: $\overline{AB}$ is a diameter of ⊙Q; $\overline{AB}$ bisects chord $\overline{CD}$

Prove: $\overline{AB} \perp \overline{CD}$

STATEMENT	REASONS
1. $\overline{AB}$ is a diameter of ⊙Q	1. Given
2. $\overline{AB}$ bisects chord $\overline{CD}$	2. Given
3. Point E is the midpoint of $\overline{CD}$	3. Definition of Line Segment Bisector
4. $\overline{CE} \cong \overline{DE}$	4. Definition of Midpoint
5. $\overline{QE} \cong \overline{QE}$	5. Reflexive Property for Congruent Line Segments
6. Draw $\overline{QC}$	6. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
7. Draw $\overline{QD}$	7. Postulate 2
8. $\overline{QC} \cong \overline{QD}$	8. Radii of the same circle are congruent
9. △QEC ≅ △QED	9. Postulate 13 - Triangle Congruence - "If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent." (SSS Congruence Assumption)
10. ∠QEC ≅ ∠QED	10. C.P.C.T.C.
11. $\overline{AB} \perp \overline{CD}$	11. Corollary 10d - "If two congruent angles form a linear pair, then the intersecting lines forming the angles are perpendicular."

2. Theorem 75 - "If a chord of a circle is a perpendicular bisector of another chord of that circle, then the original chord must be a diameter of the circle."



Given: Chord $\overline{AB}$ bisects chord $\overline{CD}$ at point X.

$\overline{AB} \perp \overline{CD}$

Prove: $\overline{AB}$ is a diameter of the circle.

STATEMENT	REASONS
1. Chord $\overline{AB}$ bisects chord $\overline{CD}$ at point X	1. Given
2. $\overline{AB} \perp \overline{CD}$	2. Given
3. Assume $\overline{AB}$ is not a diameter of ⊙Q	3. Indirect Proof Assumption
4. There must exist a chord $\overline{EF}$ through point Q, the center of the circle, and point X, the midpoint of chord $\overline{CD}$	4. Postulate 2 - For any two different points, there is exactly one line containing them.
5. $\overline{EF}$ is a diameter	5. Definition of Diameter - A line segment which is a chord of a circle, and passes through the center of that circle.
6. $\overline{EF} \perp \overline{CD}$	6. Theorem 74 - If a diameter of a circle bisects a chord that is not a diameter, then that diameter is perpendicular to that chord.
7. However, $\overline{AB} \perp \overline{CD}$ at point X	7. Given
8. $\overline{EF}$ and $\overline{AB}$ cannot both be perpendicular to $\overline{CD}$ at point X	8. Theorem 6 - If, in a plane, there is a point on a line, then there is exactly one perpendicular to the line through that point.
9. Our Assumption must be false. $\overline{AB}$ must be a diameter	9. Reductio Ad Absurdum

3. $PN = \frac{1}{2} \cdot 12$

$PN = 6$

$$(QN)^2 = (PQ)^2 + (PN)^2$$
$$(QN)^2 = (3)^2 + (6)^2$$
$$(QN)^2 = 9 + 36$$
$$(QN)^2 = 45$$
$$QN = \pm\sqrt{45} \text{ (QN cannot be negative)}$$
$$= \sqrt{9} \cdot \sqrt{5}$$
$$= 3\sqrt{5}$$

4. AB = 18 Theorem 75

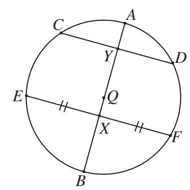
If a chord ($\overline{AB}$) of a circle is a perpendicular bisector of another chord ($\overline{MN}$) of that circle, then the original chord must be a diameter of the circle.

5. $m\angle ADB = 90$ Corollary 67a
$m\angle BED = 90$ Theorem 74
$m\widehat{AC} = 130$ Corollary 73a $m\widehat{AC} = 180 - m\widehat{BC}$
$m\angle ADC = 65$ Theorem 67

6. **a)** $\overline{RZ}$
b) point W
c) Yes, Theorem 74
d) $\overline{WX}$
e) Yes, Theorem 75

7. **a)** $m\widehat{MVN} = 180$ ($\overline{MN}$ is a diameter - Theorem 75)
b) $m\angle MVN = 90$ Corollary 67a
c) $m\angle MNV = 40$ Theorem 67
d) $m\angle NYX = 25$ $m\widehat{NV} = 100$ $m\widehat{NX} = 50$ Theorem 67
e) $m\widehat{YM} = 130$ $180 - m\widehat{NY}$

8.



Given: $\overline{AB}$ is a diameter of ⊙Q; Chord $\overline{CD}$ || Chord $\overline{EF}$; $\overline{AB}$ bisects $\overline{EF}$
Prove: $\overline{AB}$ bisects $\overline{CD}$

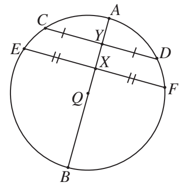
STATEMENT	REASONS
1. $\overline{AB}$ is a diameter of ⊙Q	1. Given
2. Chord $\overline{CD}$ || Chord $\overline{EF}$	2. Given
3. $\overline{AB}$ bisects $\overline{EF}$	3. Given
4. $\overline{AB} \perp \overline{EF}$	4. Theorem 74 - If a diameter of a circle bisects a chord that is not a diameter, then that diameter is perpendicular to that chord.
5. $\angle AXE$ is a right angle	5. Definition of Perpendicular Lines (Segments)
6. $m\angle AXE = 90$	6. Definition of Right Angle
7. $\angle AXE \cong \angle BYD$	7. Theorem 16a - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
8. $m\angle AXE = m\angle BYD$	8. Definition of Congruent Angles
9. $m\angle BYD = 90$	9. Substitution
10. $\angle BYD$ is a right angle	10. Definition of Right Angle
11. $\overline{AB} \perp \overline{CD}$	11. Definition of Perpendicular Line (Segments)
12. $\overline{AB}$ bisects $\overline{CD}$	12. Theorem 73 - If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects that chord.

9.

STATEMENT	REASONS
1. $\overline{AB}$ is a diameter of ⊙Q	1. Given
2. $\overline{CE} \cong \overline{DE}$	2. Given
3. $\overline{AB}$ bisects $\overline{CD}$	3. Definition of Segment Bisector
4. $\overline{AB} \perp \overline{CD}$	4. Theorem 74 - If a diameter of a circle bisects a chord that is not a diameter, then that diameter is perpendicular to that chord.
5. $\angle AEC \cong \angle AED$	5. Corollary 10c - If two lines are perpendicular, then they form congruent adjacent angles.
6. $\overline{AE} \cong \overline{AE}$	6. Reflexive Property for Congruent Line Segments
7. $\triangle AEC \cong \triangle AED$	7. Postulate 13 - Triangle Congruence - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle then the two triangles are congruent. (SAS Congruence Assumption)

10. Infinitely many; one
11. Diameter; one; two
12. A secant
13. 18; The chord is a diameter - Corollary 68a
14. The center of the circle

15. Given: $\overline{AB}$ is a diameter of $\odot Q$
$\overline{AB}$ bisects $\overline{CD}$; $\overline{AB}$ bisects $\overline{EF}$
Prove: $\overline{CD} \parallel \overline{EF}$



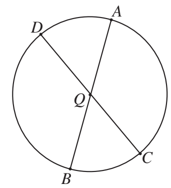
STATEMENT	REASONS
1. $\overline{AB}$ is a diameter of $\odot Q$	1. Given
2. $\overline{AB}$ bisects $\overline{CD}$	2. Given
3. $\overline{AB} \perp \overline{CD}$	3. Theorem 74 - If a diameter of a circle bisects a chord that is not a diameter, then that diameter is perpendicular to that chord.
4. $\overline{AB}$ bisects $\overline{EF}$	4. Given
5. $\overline{AB} \perp \overline{EF}$	5. Theorem 74
6. $\overline{CD} \parallel \overline{EF}$	6. Theorem 23 - If two lines are perpendicular to a third line, then the two lines are parallel.

16. Two chords of a circle that are not diameters are parallel to each other if and only if a diameter of the circle bisects the two chords.

17. Proof of Theorem 75

STATEMENT	REASONS
1. Chord $\overline{AB}$ bisects chord $\overline{CD}$ at point E	1. Given
2. Point E is the midpoint of $\overline{CD}$	2. Definition of Line Segment Bisector
3. $\overline{CE} \cong \overline{DE}$ at point E	3. Definition of Midpoint
4. $\overline{AB} \perp \overline{CD}$	4. Given
5. $\angle AEC$ is a right angle	5. Definition of Perpendicular Lines (Segments)
6. $\angle AED$ is a right angle	6. Definition of Perpendicular Lines (Segments)
7. $\angle AEC \cong \angle AED$	7. Theorem 11 - If you have right angles, then those right angles are congruent.
8. $\overline{AE} \cong \overline{AE}$	8. Reflexive Property for Congruent Line Segments
9. Draw $\overline{AC}$	9. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
10. Draw $\overline{AD}$	10. Postulate 2
11. $\triangle AEC \cong \triangle AED$	11. Postulate 13 - Triangle Congruence - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle then the two triangles are congruent. (SAS Congruence Assumption)
12. $\angle CAB \cong \angle DAB$	12. C.P.C.T.C.
13. $m\angle CAB = \frac{1}{2} m\overset{\frown}{BC}$	13. Theorem 67 - If you have an inscribed angle of a circle, then the measure of the angle is one-half the measure of the intercepted arc.
14. $m\angle DAB = \frac{1}{2} m\overset{\frown}{BD}$	14. Theorem 67
15. $m\angle CAB = m\angle DAB$	15. Definition of Congruent Angles
16. $\frac{1}{2} m\overset{\frown}{BC} = \frac{1}{2} m\overset{\frown}{BD}$	16. Substitution
17. $m\overset{\frown}{BC} = m\overset{\frown}{BD}$	17. Multiplication Property of Equality
18. $\angle ACD \cong \angle ADC$	18. C.P.C.T.C.
19. $m\angle ACD = \frac{1}{2} m\overset{\frown}{DA}$	19. Theorem 67
20. $m\angle ADC = \frac{1}{2} m\overset{\frown}{AC}$	20. Theorem 67
21. $m\angle ACD = m\angle ADC$	21. Definition of Congruent Angles
22. $\frac{1}{2} m\overset{\frown}{DA} = \frac{1}{2} m\overset{\frown}{AC}$	22. Substitution
23. $m\overset{\frown}{DA} = m\overset{\frown}{AC}$	23. Multiplication Property of Equality
24. $m\overset{\frown}{BC} + m\overset{\frown}{BD} + m\overset{\frown}{DA} + m\overset{\frown}{AC} = 360$	24. Postulate 8 - First Assumption - The set of all points on a circle can be put into a one-to-one correspondence with the real numbers from 0 to 360, inclusive, with the exception of any one point which may be paired with 0 and 360.
25. $m\overset{\frown}{BC} + m\overset{\frown}{BC} + m\overset{\frown}{AC} + m\overset{\frown}{AC} = 360$	25. Substitution
26. $2m\overset{\frown}{BC} + 2m\overset{\frown}{AC} = 360$	26. Properties of Algebra - Collect Like Terms
27. $m\overset{\frown}{BC} + m\overset{\frown}{AC} = 180$	27. Multiplication Property of Equality
28. $m\overset{\frown}{BC} + m\overset{\frown}{AC} = m\overset{\frown}{ACB}$	28. Postulate 8 - Fourth Assumption - Arc Addition Assumption
29. $m\overset{\frown}{ACB} = 180$	29. Substitution
30. $\overset{\frown}{ACB}$ is a semicircle	30. Definition of Semicircle - "...a semicircle is the intercepted arc of a central angle of 180^{o}"
31. $\overline{AB}$ is a diameter of the circle	31. Definition of Semicircle - An arc of a circle is a semicircle, if and only if, it is an arc whose endpoints are the endpoints of a diameter of a circle.

18. $\overline{AB}$ is a diameter. The midpoint of any diameter is the center of the circle. In ⊙Q, $\overline{AB}$ bisects $\overline{CD}$. Since $\overline{CD}$ is also a diameter, $\overline{AB}$ can bisect $\overline{CD}$ and not be perpendicular to $\overline{CD}$.

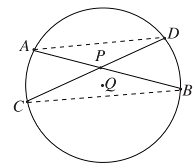


Unit VI — Circles

Part C — Line and Segment Relationships

p. 563 – Lesson 3 — Theorem 76

1. Theorem 76 - "If two chords intersect within a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord."



Given: $\overline{AB}$ and $\overline{CD}$ are chords of ⊙Q intersecting at point P.
Prove: AP • PB = CP • PD

STATEMENT	REASONS
1. $\overline{AB}$ and $\overline{CD}$ are chords of ⊙Q intersecting at point P	1. Given
2. Draw chord $\overline{AD}$	2. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
3. Draw chord $\overline{CB}$	3. Postulate 2
4. $\angle A \cong \angle C$	4. Corollary 67c - If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.
5. $\angle D \sim \angle B$	5. Corollary 67c
6. $\triangle APD \cong \triangle CPB$	6. Postulate Corollary 12a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar.
7. $\frac{AP}{CP} = \frac{PD}{PB}$	7. Definition of Similarity - Two polygons are similar if and only if corresponding angles are congruent and corresponding sides are in proportion.
8. AP • PB = CP • PD	8. Multiplication Property of Equality (Multiply both sides by CP • PB)

2.
$$(4)(x) = (8)(5)$$
$$4x = 40$$
$$\frac{1}{4}\cdot 4x = \frac{1}{4}\cdot 40$$
$$\frac{1\cdot \cancel{4}\cdot x}{\cancel{4}} = \frac{1\cdot \cancel{4}\cdot 10}{\cancel{4}}$$
$$x = 10$$

3.
$$(x)(x) = (18)(8)$$
$$x^2 = 144$$
$$x^2 - 144 = 0$$
$$(x-12)(x+12) = 0$$
$$x - 12 = 0$$
$$\text{or } x + 12 = 0$$

$x = 12$
or $x = -12$ (not possible)
so, $x = 12$

4.
$$(x)(x+6) = (4)(12)$$
$$x^2 + 6x = 48$$
$$x^2 + 6x - 48 = 0$$

$a = 1 \quad b = 6 \quad c = -48$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-48)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{36 + 192}}{2}$$
$$= \frac{-6 \pm \sqrt{228}}{2}$$
$$= \frac{-6 \pm \sqrt{4\cdot 57}}{2}$$
$$= \frac{-6 \pm 2\sqrt{57}}{2}$$
$$= \frac{\cancel{2}\left(-3 \pm \sqrt{57}\right)}{\cancel{2}}$$
$$= -3 + \sqrt{57} \text{ or } -3 - \sqrt{57} \ (\text{x cannot be negative})$$
$$= -3 + \sqrt{57} \text{ or approximately } 4.5498$$

5.
$$(4x)(x) = (8)(6)$$
$$4x^2 = 48$$
$$x^2 = 12$$
$$x = \pm\sqrt{12}$$
$$x = \sqrt{12}$$
$$\text{or } x = -\sqrt{12} \text{ (not possible)}$$

$$\text{so, } x = \sqrt{4\cdot 3}$$
$$x = \sqrt{4}\cdot\sqrt{3}$$
$$x = 2\sqrt{3}$$

6. $(8-x)(x)=(4)(3)$

$$8x-x^2=12$$
$$0=x^2-8x+12$$
$$0=(x-6)(x-2)$$
$$0=x-6$$
$$\text{or } 0=x-2$$
$$x=6$$
$$\text{or } x=2$$

If x = 6, then 8 – x = 2.
If x = 2, then 8 – 2 = 6.

7. $(x)(12-x)=(4)(5)$

$$12x-x^2=20$$
$$0=x^2-12x+20$$
$$0=(x-2)(x-10)$$
$$0=x-2$$
$$\text{or } 0=x-10$$
$$x=2$$
$$\text{or } x=10$$

If x = 2, then 12 – x = 10.
If x = 10, then 12 – x = 2.

8. $(x+2)(2x-1)=x(x+4)$

$$2x^2+3x-2=x^2+4x$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x-2=0$$
$$\text{or } x+1=0$$
$$x=2$$
$$\text{or } x=-1 \text{ (cannot be negative)}$$

So, $x=2$

9. $AQ\cdot BQ=DQ\cdot CQ$

$$AQ\cdot 5=6\cdot 7$$
$$5\cdot AQ=42$$
$$AQ=\frac{42}{5}$$

$$AQ+BQ=AB$$
$$\frac{42}{5}+5=AB$$
$$\frac{42}{5}+\frac{25}{5}=AB$$
$$\frac{67}{5}=AB$$

10. $CE\cdot DE=FE\cdot AE$

$$3\cdot 3=(FB+BE)\cdot(AB-BE)$$
$$9=(r+4)(r-4)$$
$$9=r^2-16$$
$$0=r^2-25$$
$$0=(r+5)(r-5)$$
$$0=r+5$$
$$\text{or } 0=r-5$$
$$r=-5 \text{ (r cannot be negative)}$$
$$\text{or } r=5$$

$$AB=r$$
$$AB=5$$

11. MX, NX, PX, and QX where point X is the intersection of $\overline{MN}$ and $\overline{PQ}$

12. Theorem 76 - (MX)(NX) must equal (PX)(QX).

13. $\overline{MP}$ is a diameter of the circle of $\overset{\frown}{XMY}$. (Theorem 75)
XN = 22
YN = 22
MN = 120
Diameter $\overline{MP}=2\cdot r$ Therefore, NP = 2r – 120

By Theorem 76, $(XN)(YN)=(MN)(NP)$

$$(22)(22)=(120)(2r-120)$$

mult. by $\frac{1}{4}$ $$\frac{\cancel{2}\cdot 11\cdot \cancel{2}\cdot 11}{\cancel{2}\cdot\cancel{2}}=\frac{\cancel{2}\cdot 60\cdot\cancel{2}\cdot(r-60)}{\cancel{2}\cdot\cancel{2}}$$

$$121=60r-3600$$
$$3721=60r$$

mult. by $\frac{1}{60}$ $$\frac{3721}{60}=r$$

$$62\frac{1}{60}=r$$

62 feet is approximately the radius.

14.

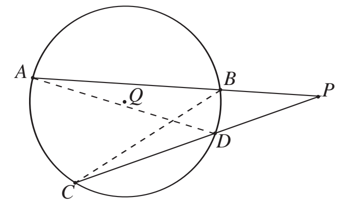
$$r=\frac{\left(\frac{1}{2}w\right)^2+h^2}{2h}$$
$$=\frac{\left(\frac{1}{2}\cdot 200\right)^2+40^2}{2\cdot 40}$$
$$=\frac{(100)^2+40^2}{80}$$
$$=\frac{10{,}000+1600}{80}$$
$$=\frac{11{,}600}{80}$$
$$=\frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{5}\cdot 145}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{5}}$$
$$=145$$

Unit VI — Circles

Part C — Line and Segment Relationships

p. 566 – Lesson 4 — Theorem 77 & 78

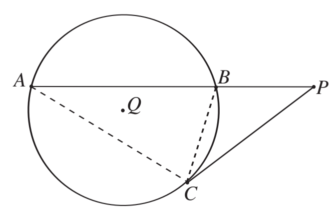
1. Theorem 77 - "If two secant segments are drawn to a circle from a single point outside the circle, the product of the lengths of one secant segment and its external segment, is equal to the product of the lengths of the other secant segment and its external segment."



Given: $\overline{PA}$ and $\overline{PC}$ are secants of ⊙Q.
Prove: AP • BP = CP • DP

STATEMENT	REASONS
1. $\overline{PA}$ and $\overline{PC}$ are secants of ⊙Q	1. Given
2. Draw $\overline{AD}$	2. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
3. Draw $\overline{CB}$	3. Postulate 2
4. ∠A ≅ ∠C	4. Corollary 67c - If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.
5. ∠P ≅ ∠P	5. Reflexive Property of Angle Congruence
6. △APD ~ △CPB	6. Postulate Corollary 12a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar. (AA)
7. $\frac{AP}{CP} = \frac{DP}{BP}$	7. Definition of Similarity - Two polygons are similar if and only if corresponding angles are congruent and corresponding sides are in proportion.
8. AP • BP = CP • DP	8. Multiplication Property of Equality (Multiply both sides by CP • PB)

2. Theorem 78 - "If a secant segment and a tangent segment are drawn to a circle from a single point outside the circle, then the length of the tangent segment is the mean proportional between the length of the secant segment and its external segment."



Given: $\overline{PA}$ is a secant segment of ⊙Q.
$\overline{PC}$ is a tangent segment to ⊙Q.
Prove: $\frac{AP}{CP} = \frac{CP}{BP}$ ($\overline{CP}$ mean proportional between $\overline{AP}$ and $\overline{BP}$.)

STATEMENT	REASONS
1. $\overline{PA}$ is a secant segment of ⊙Q.	1. Given
2. $\overline{PC}$ is a tangent segment to ⊙Q.	2. Given
3. Draw chord $\overline{AC}$	3. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
4. Draw chord $\overline{BC}$	4. Postulate 2
5. $m\angle PCB = \frac{1}{2} m\overset{\frown}{BC}$	5. Theorem 68 - If you have an angle formed by a secant and a tangent at the point of tangency, then the measure of that angle is one-half the measure of its intercepted arc.
6. $m\angle PAC = \frac{1}{2} m\overset{\frown}{BC}$	6. Theorem 67 - If an angle is inscribed in a circle, then the measure of that angle is one-half the measure of the intercepted arc.
7. m∠PCB = m∠PAC	7. Substitution
8. ∠PCB ≅ ∠PAC	8. Definition of Congruent Angles
9. ∠P ≅ ∠P	9. Reflexive Property for Congruent Angles
10. △APD ~ △CPB	10. Postulate Corollary 12a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar. (AA)
11. $\frac{AP}{CP} = \frac{CP}{BP}$	11. Definition of Similarity - Two polygons are similar if and only if corresponding angles are congruent and corresponding sides are in proportion.
12. $\overline{CP}$ is the mean proportional between $\overline{AP}$ and $\overline{BP}$	12. Definition of Mean Proportional

3. $(5+3)\cdot 3=(4+x)\cdot 4$

$8\cdot 3=16+4x$

$24=16+4x$

$8=4x$

$2=x$

4. $\frac{3+4}{x}=\frac{x}{3}$

$\frac{7}{x}=\frac{x}{3}$

$x^2=21$

$x=\pm\sqrt{21}$

$x=\sqrt{21}$ (x cannot be negative)

5. $(6+4)\cdot 6=x\cdot 5$

$10\cdot 6=5x$

$60=5x$

$12=x$

6. $\frac{9}{6}=\frac{6}{x}$

$6\cdot 6=9\cdot x$

$36=9x$

$4=x$

7. $(7+5)\cdot 5=10\cdot x$

$12\cdot 5=10x$

$60=10x$

$6=x$

8. $\frac{x+3x}{10}=\frac{10}{x}$

$\frac{4x}{10}=\frac{10}{x}$

$10\cdot 10=(4x)(x)$

$100=4x^2$

$25=x^2$

$\pm\sqrt{25}=x$

$\pm 5=x$

$5=x$ (x cannot be negative)

9. $\frac{4+7}{x}=\frac{x}{4}$

$x\cdot x=(4+7)\cdot 4$

$x^2=11\cdot 4$

$x^2=44$

$x=\pm\sqrt{44}$ (x cannot be negative)

$x=\sqrt{44}$

$x=\sqrt{4\cdot 11}$

$x=2\sqrt{11}$

10. $\left[(x+1)+x\right]\cdot(x+1)=\left[5+(2x-4)\right]\cdot 5$

$(2x+1)(x+1)=(2x+1)\cdot 5$

$2x^2+3x+1=10x+5$

$2x^2-7x-4=0$

$(2x+1)(x-4)=0$

$2x+1=0$ or $x-4=0$

$2x=-1$ $x=4$

$x=-\frac{1}{2}$

x cannot be negative

11. $(12)(3)=(8)(x)$

$36=8x$

$\frac{36}{8}=x$

$\frac{\not{2}\cdot\not{2}\cdot 3\cdot 3}{\not{2}\cdot\not{2}\cdot 2}=x$

$\frac{9}{2}=x$

12. $\frac{3+x}{9}=\frac{9}{3}$

$9\cdot 9=(3+x)\cdot 3$

$81=9+3x$

$72=3x$

$\frac{72}{3}=x$

$\frac{9\cdot 8}{3}=x$

$\frac{\not{3}\cdot 3\cdot 8}{\not{3}}=x$

$24=x$

13. $\frac{a+b}{x}=\frac{x}{a}$

$x\cdot x=(a+b)\cdot a$

$x^2=a^2+ab$

$x=\pm\sqrt{a^2+ab}$ (x cannot be negative)

$x=\sqrt{a^2+ab}$

14. $3(3+5x)=x(x+2x)$

$9+15x=x^2+2x^2$

$9+15x=3x^2$

$0=3x^2-15x-9$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-15)\pm\sqrt{(-15)^2-4(3)(-9)}}{2(3)}$

$x=\frac{15\pm\sqrt{225+108}}{6}$

$x=\frac{15\pm\sqrt{333}}{6}$

$x=\frac{15\pm\sqrt{9\cdot 37}}{6}$

$x=\frac{15\pm 3\sqrt{37}}{6}$

$x=\frac{\not{3}\left(5\pm\sqrt{37}\right)}{\not{3}\cdot 2}$

$x=\frac{5\pm\sqrt{37}}{2}$ (x cannot be negative)

$x=\frac{5+\sqrt{37}}{2}$

15. $\frac{x+9}{6}=\frac{6}{x}$

$(6)(6)=(x+9)(x)$

$36=x^2+9x$

$0=x^2+9x-36$

$0=(x+12)(x-3)$

$0=x+12$

or $0=x-3$

$-12=x$ (cannot be negative)

or $3=x$

$(4+y)\cdot 4=(3+9)\cdot 3$

$16+4y=12\cdot 3$

$16+4y=36$

$4y=20$

$y=5$

16. $\frac{6+x}{8}=\frac{8}{6}$

$(8)(8)=(6+x)(6)$

$64=36+6x$

$28=6x$

$\frac{28}{6}=x$

$\frac{\cancel{2}\cdot 2\cdot 7}{\cancel{2}\cdot 3}=x$

$\frac{14}{3}=x$

$(5+x)\cdot 5=(4+y)\cdot 4$

$\left(5+\frac{14}{3}\right)\cdot 5=16+4y$

$\left(\frac{15}{3}+\frac{14}{3}\right)\cdot 5=16+4y$

$\left(\frac{29}{3}\right)\cdot 5=16+4y$

$\frac{145}{3}=16+4y$

$\frac{145}{3}-16=4y$

$\frac{145}{3}-\frac{48}{3}=4y$

$\frac{97}{3}=4y$

$\frac{1}{4}\cdot\frac{97}{3}=y$

$\frac{97}{12}=y$

17. a) $\frac{PA}{PT}=\frac{PT}{PB}$

$\frac{PA}{6}=\frac{6}{3}$

$3(PA)=(6)(6)$

$3(PA)=36$

$PA=\frac{36}{3}$

$PA=\frac{2\cdot 3\cdot 2\cdot \cancel{3}}{\cancel{3}}$

$PA=12$

$PA-PB=AB$

$12-3=AB$

$9=AB$

b) $\frac{PC}{PT}=\frac{PT}{PD}$

$\frac{CD+PD}{PT}=\frac{PT}{PD}$

$\frac{18+PD}{12}=\frac{12}{PD}$

$(12)(12)=(18+PD)(PD)$

$144=18(PD)+PD^2$

$0=PD^2+18PD-144$

$0=(PD-6)(PD+24)$

$0=PD-6$

or $0=PD+24$

$PD=6$

or $PD=-24$

(PD cannot be negative)

$PD+CD=PC$

$6+18=PC$

$24=PC$

c) $(PC)(PD)=(PA)(PB)$

$(PD+DC)(PD)=(AB+PB)(PB)$

$(5+7)(5)=(11+PB)(PB)$

$(12)(5)=11(PB)+PB^2$

$60=11PB+PB^2$

$0=PB^2+11PB-60$

$0=(PB+15)(PB-4)$

$0=PB+15$

or $0=PB-4$

$PB=-15$

(PB cannot be negative)

$PB=4$

d) $\frac{PA}{PT}=\frac{PT}{PB}$

$\frac{PB+AB}{PT}=\frac{PT}{PB}$

$\frac{5+5}{PT}=\frac{PT}{5}$

$\frac{10}{PT}=\frac{PT}{5}$

$PT^2=(10)(5)$

$PT^2=50$

$PT=\pm\sqrt{50}$

(PT cannot be negative)

$PT=\sqrt{50}$

$PT=\sqrt{25\cdot 2}$

$PT=5\sqrt{2}$

$(PC)(PD)=(PA)(PB)$

$(PC)(4)=(PB+AB)(PB)$

$(PC)(4)=(5+5)(5)$

$(PC)(4)=(10)(5)$

$PC=\frac{10\cdot 5}{4}$

$PC=\frac{\cancel{2}\cdot 5\cdot 5}{\cancel{2}\cdot 2}$

$PC=\frac{25}{2}$

18. $(RT)^2 + (PT)^2 = (RP)^2$

$3^2 + (PT)^2 = 8^2$

$9 + PT^2 = 64$

$PT^2 = 55$

$PT = \pm\sqrt{55}$

(PT cannot be negative)

$PT = \sqrt{55}$

a) $\frac{TS}{PT} = \frac{PT}{TR}$

$\frac{TR + RS}{PT} = \frac{PT}{TR}$

$\frac{3 + RS}{\sqrt{55}} = \frac{\sqrt{55}}{3}$

$(\sqrt{55})(\sqrt{55}) = (3 + RS)(3)$

$55 = 9 + 3RS$

$46 = 3RS$

$\frac{46}{3} = RS$

b) The distance from point Q to $\overleftrightarrow{RS}$ is the perpendicular from point Q to $\overleftrightarrow{RS}$. SInce $\overline{PT} \perp \overleftrightarrow{RS}$ at point T and $\overline{PQ} \perp \overline{PT}$ at point P (Corollary 68a), a rectangle is formed. PT is the same measure as the distance from point Q to $\overleftrightarrow{RS}$. Therefore, $PT = \sqrt{55}$.

c) $RS = \frac{46}{3}$

The perpendicular from point Q to $\overleftrightarrow{RS}$ also bisects $\overline{RS}$. (Theorem 73) Therefore, $\frac{1}{2}RS = \frac{1}{2} \cdot \frac{46}{3} = \frac{\cancel{2} \cdot 23}{\cancel{2} \cdot 3} = \frac{23}{3}$

$RT + \frac{1}{2}RS =$ radius PQ since the sides of a rectangle are equal.

$3 + \frac{23}{3} = PQ$

$\frac{9}{3} + \frac{23}{3} = PQ$

$\frac{32}{3} = PQ$

19. a) The triangle shown is a right triangle since a radius is perpendicular to a tangent. (Corollary 68a). The short leg of the right triangle is a radius. The Pythagorean Theorem applies: $(\text{Hypotenuse})^2 = (\text{short leg})^2 + (\text{long leg})^2$ or $(r + u)^2 = r^2 + d^2$

b) $(r + u)^2 = r^2 + d^2$

$(r + u)(r + u) = r^2 + d^2$

$r^2 + 2ru + u^2 = r^2 + d^2$

$2ru + u^2 = d^2$

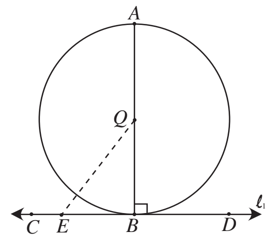
$u(2r + u) = d^2$

c) Theorem 78

Unit VI — Circles
Part C — Line and Segment Relationships

p. 570 – Lesson 5 — Theorem 79

1. Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."



Given: $\odot Q$ with diameter $\overline{AB}$.
$\overleftrightarrow{CD} \perp \overline{AB}$ at point B
Prove: $\overleftrightarrow{CD}$ is tangent to $\odot Q$.

STATEMENT	REASONS
1. $\odot Q$ with diameter $\overline{AB}$	1. Given
2. $\overleftrightarrow{CD} \perp \overline{AB}$ at point B	2. Given
3. Choose a point E on $\overleftrightarrow{CD}$ and draw $\overline{QE}$.	3. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
4. QB < QA	4. A segment is the shortest segment from a point to a line if and only it it is the segment perpendicular to the line.
5. Point E lies in the exterior of $\odot Q$	5. If a point is in the exterior of a circle, then the measure of the segment joining the point to the center of the circle is greater than the measure of the radius.
6. $\overleftrightarrow{CD}$ is tangent to $\odot Q$	6. Definition of Tangent

2. Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."

Given: Line *m* is perpendicular to $\overline{PA}$ at point A.
Prove: Line *m* is tangent to $\odot Q$.

Proof: Let point B be any point on line *m* other than point A. SInce $\overline{PA} \perp m$, $\triangle QAB$ is a right triangle with hypotenuse $\overline{QB}$. This means QB > QA and point B must be in the exterior of $\odot Q$. Therefore, point B cannot lie on the circle and point A is the only point on line *m* that is on the circle. It follows that line *m* is tangent to the circle.

3. 90 degrees

4. Tangent; Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."

5. 6

6. 42 degrees

7. $3\sqrt{3}$

$$\frac{3}{FG} = \frac{FG}{9}$$
$$(FG)^2 = 27$$
$$FG = \pm\sqrt{27} \text{ (FG cannot be negative)}$$
$$FG = \sqrt{9 \cdot 3}$$
$$FG = 3\sqrt{3}$$

8. 35

$$JP + PI + IF + FH + HQ + QK$$
$$6 + 6 + 2 + 3 + 9 + 9$$
$$12 + 5 + 18$$
$$35$$

9. 48 degrees

10. Tangent; Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."

11. $2\sqrt{7}$

$$(PF)^2 = (FE)^2 + (PE)^2$$
$$(8)^2 = (FE)^2 + (6)^2$$
$$64 = (FE)^2 + 36$$
$$28 = (FE)^2$$
$$\pm\sqrt{28} = FE \text{ (FE cannot be negative)}$$
$$\sqrt{4 \cdot 7} = FE$$
$$2\sqrt{7} = FE$$

12. $\sqrt{391}$ Draw $\overline{PM} \perp$ to $\overline{QC}$ forming right $\triangle PMQ$. $\overline{PM} \cong \overline{DC}$.

$$(PM)^2 + (MQ)^2 = (PQ)^2$$
$$(PM)^2 + (9-6)^2 = (6+2+3+9)^2$$
$$(PM)^2 + (3)^2 = (20)^2$$
$$(PM)^2 + 9 = 400$$
$$(PM)^2 = 391$$
$$PM = \pm\sqrt{391} \text{ (PM cannot be negative)}$$
$$PM = \sqrt{391}$$

13. 42 degrees

14. 8 6 + 2

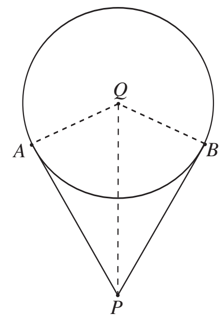
15. P; Q Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."

16. In the plane of ⊙Q, the line *m* is tangent to ⊙Q at point A, if and only if, the line *m* is perpendicular to diameter $\overline{PA}$ at point A.

Unit VI — Circles

Part C — Line and Segment Relationships

p. 572 – Lesson 6 — Theorem 80 & Corollary 80a



1. Theorem 80 - "If two tangent segments are drawn to a circle from the same point, then those segments are congruent."

Given: $\overline{PA}$ and $\overline{PB}$ are tangent to ⊙Q, drawn from point P.
Prove: $\overline{PA} \cong \overline{PB}$

STATEMENT	REASONS
1. $\overline{PA}$ and $\overline{PB}$ are tangent to ⊙Q, drawn from point P	1. Given
2. Draw $\overline{PQ}$	2. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
3. Draw $\overline{BQ}$	3. Postulate 2
4. Draw $\overline{AQ}$	4. Postulate 2
5. $\overline{PA} \perp \overline{QA}$	5. Corollary 68a - If, in a circle, a diameter (radius) is drawn to a tangent line at the point of tangency, then the diameter (radius) is perpendicular to the tangent line at that point.
6. $\overline{PB} \perp \overline{QB}$	6. Corollary 68a
7. ∠QAP is a right angle	7. Definition of Perpendicular Lines (segments)
8. ∠QBP is a right angle	8. Definition of Perpendicular Lines (segments)
9. △QAP is a right triangle	9. Definition of Right Triangle
10. △QBP is a right triangle	10. Definition of Right Triangle
11. $\overline{QA} \cong \overline{QB}$	11. Radii of the same circle are congruent
12. $\overline{PQ} \cong \overline{PQ}$	12. Reflexive Property for congruent line segments
13. △QAP ≅ △QBP	13. Postulate Corollary 13c - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two right triangles are congruent. (HL)
14. $\overline{PA} \cong \overline{PB}$	14. C.P.C.T.C.

2. If UV = 3, then YV = 3. (Theorem 80)
If XV = 5, then WV = 5. (Theorem 80)

$$\begin{aligned} UV + WV &= UW \\ 3 + 5 &= UW \\ 8 &= UW \end{aligned}$$

$$\begin{aligned} YV + XV &= XY \\ 3 + 5 &= XY \\ 8 &= XY \end{aligned}$$

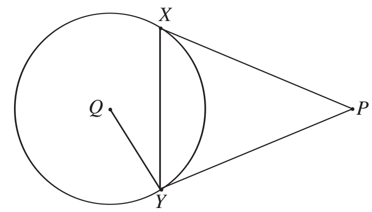
3. If PA = 12, then PB = 12. (Theorem 80)
If PB = 12, then PC = 12. (Theorem 80)
If PC = 12, then PD = 12. (Theorem 80)

4. Given: $\angle 1 \cong \angle 2 \cong \angle 3$
Prove: $\triangle ABC$ is equilateral

STATEMENT	REASONS
1. $\angle 1 \cong \angle 2 \cong \angle 3$	1. Given
2. $m\angle 1 = m\angle 2 = m\angle 3$	2. Definition of Congruent Angles
3. $m\angle 1 = m\overset{\frown}{DF}$; $m\angle 2 = m\overset{\frown}{FE}$; $m\angle 3 = m\overset{\frown}{ED}$	3. The measure of an arc of a circle is equal to the measure of its central angle.
4. $m\overset{\frown}{DF} = m\overset{\frown}{FE} = m\overset{\frown}{ED}$	4. Substitution
5. $m\overset{\frown}{DF} + m\overset{\frown}{FE} + m\overset{\frown}{ED} = 360$	5. Postulate 8 - Circle - The set of all points on a circle can be put into a one-to-one correspondence with the real numbers from 0 to 360, inclusive, with the exception of any one point which may be paired with 0 and 360.
6. $m\overset{\frown}{DF} + m\overset{\frown}{DF} + m\overset{\frown}{DF} = 360$	6. Substitution
7. $3m\overset{\frown}{DF} = 360$	7. Properties of Arithmetic; Collect Like Terms
8. $m\overset{\frown}{DF} = 120$	8. Multiplication Property of Equality
9. $m\angle DQF = 120$	9. The measure of a central angle is equal to the measure of its intercepted arc.
10. $m\overset{\frown}{FE} + m\overset{\frown}{FE} + m\overset{\frown}{FE} = 360$	10. Substitution
11. $3m\overset{\frown}{FE} = 360$	11. Properties of Arithmetic; Collect Like Terms
12. $m\overset{\frown}{FE} = 120$	12. Multiplication Property of Equality
13. $m\angle FQE = 120$	13. The measure of a central angle is equal to the measure of its intercepted arc.
14. $m\overset{\frown}{ED} + m\overset{\frown}{ED} + m\overset{\frown}{ED} = 360$	14. Substitution
15. $3m\overset{\frown}{ED} = 360$	15. Properties of Arithmetic; Collect Like Terms
16. $m\overset{\frown}{ED} = 120$	16. Multiplication Property of Equality
17. $m\angle DQE = 120$	17. The measure of a central angle is equal to the measure of its intercepted arc.
18. $\overline{QD} \perp \overline{AC}$	18. Corollary 68a - If, in a circle, a diameter (radius) is drawn to a tangent line at the point of tangency, then the diameter (radius) is perpendicular to the tangent line at that point.
19. $\overline{QE} \perp \overline{AB}$	19. Corollary 68a
20. $\overline{QF} \perp \overline{BC}$	20. Corollary 68a
21. $\angle ADQ$ is a right angle	21. Definition of Perpendicular
22. $\angle CDQ$ is a right angle	22. Corollary 10a - If one angle of a linear pair is a right angle then the other angle is a right angle.
23. $\angle AEQ$ is a right angle	23. Definition of Perpendicular
24. $\angle BEQ$ is a right angle	24. Corollary 10a
25. $\angle BFQ$ is a right angle	25. Definition of Perpendicular
26. $\angle CFQ$ is a right angle	26. Corollary 10a
27. $m\angle ADQ = m\angle CDQ = m\angle AEQ = m\angle BEQ = m\angle BFQ = m\angle CFQ = 90$	27. Definition of Right Angle
28. $m\angle DQE + m\angle ADQ + m\angle AEQ + m\angle DAE = 360$	28. The sum of the measures of the angles of a quadrilateral is 360^{o}.
29. $120 + 90 + 90 + m\angle DAE = 360$	29. Substitution
30. $m\angle DAE = 60$	30. Subtraction Property of Equality
31. $m\angle FQE + m\angle BEQ + m\angle BFQ + m\angle EBF = 360$	31. The sum of the measures of the angles of a quadrilateral is 360^{o}.
32. $120 + 90 + 90 + m\angle EBF = 360$	32. Substitution
33. $m\angle EBF = 60$	33. Subtraction Property of Equality
34. $m\angle DQF + m\angle CDQ + m\angle CFG + m\angle DCF = 360$	34. The sum of the measures of the angles of a quadrilateral is 360^{o}.
35. $120 + 90 + 90 + m\angle DCF = 360$	35. Substitution
36. $m\angle DCF = 60$	36. Subtraction Property of Equality
37. $\angle DAE \cong \angle EBF \cong \angle DCF$	37. Definition of Congruent Angles
38. $\triangle ABC$ is equiangular	38. A triangle is an equiangular triangle, if and only if, all of its angles are congruent.
39. $\triangle ABC$ is equilateral	39. Corollary 34a - Equiangular triangles are equilateral - Q.E.D.

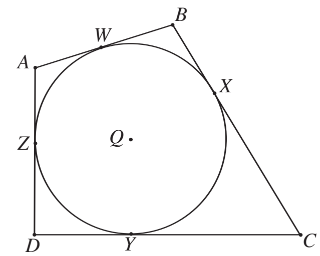
A paragraph argument: It is given that $\angle 1 \cong \angle 2 \cong \angle 3$, so by definition $m\angle 1 = m\angle 2 = m\angle 3$. Since a circle contains 360 degrees in one revolution, $m\angle 1 + m\angle 2 + m\angle 3 = 360$, and each angle measures 120 degrees. The angles formed by the three tangents with the three radii are all right angles, so we have three quadrilaterals with 300 degrees in each plus an angle of the triangle in each to equal 360. Therefore, each angle of $\triangle ABC$ must equal 60 degrees, and the triangle is equiangular and also equilateral.

5. $\overline{PX} \cong \overline{PY}$ (Theorem 80)
$\triangle$PYX is isosceles by definition of an isosceles triangle (two sides are congruent).
$\angle$PYZ $\cong$ $\angle$PXY (Theorem 33)
$\overline{QY} \perp \overline{PY}$ (Corollary 68a)
$\angle$QYP is a right angle, so m$\angle$QYP = 90.
SInce m$\angle$XYQ = 10 degrees, m$\angle$XYP = 80 degrees. (90 - 10 = 80)
The sum of the measures of the angles of a triangle is 180.



$$\begin{aligned} m\angle XYP + m\angle YXP + m\angle P &= 180 \\ 80 + 80 + m\angle P &= 180 \\ 160 + m\angle P &= 180 \\ m\angle P &= 20 \end{aligned}$$

6. Label the points of tangency W, X, Y, and Z.



Part 1:
AW = AZ, BW = BX, CX = CY and DY = DZ (Theorem 80)
AB = AW + BW and DC = DY + CY.
AB + DC = AW + BW + DY + CY

Part 2:
AD = AZ + DZ and BC = BX + CX
AD + BC = AZ + DZ + BX + CX
Substituting AW for AZ, BW for BX, DY for DZ, and CY for CX, we have
AD + BC = AW + DY + BW + CY
Using the Commutative Property for Addition, we have
AD +BC = AW + BW + DY + CY
Therefore, AB + DC = AD + BC

7. RX = RZ and TY = TZ (Theorem 80)
PX = PR + RX
PY = PT + TY
PX + PY = PR + RX + PT + TY
RT = RZ + TZ
RT = RX +TY (Substituting RX for RZ and TY for TZ)
Since PZ + PY = PR + RX + TY + PT (Using the Commutative Property of Addition)
we can substitute RT for RX +TY to get
PX + PY = PR + RT + PT

8. $\overline{QB} \perp \overline{AB}$ and $\overline{QC} \perp \overline{AC}$ (Corollary 68a)
$\angle$QBA and $\angle$QCA are right angles.

$$\begin{aligned} m\angle QBA = m\angle QCA &= 90 \\ m\angle QBA + m\angle BQA + y &= 180 \\ 90 + 80 + y &= 180 \\ 170 + y &= 180 \\ y &= 10 \end{aligned}$$

Therefore, W = 10 (Corollary 80a)

$$\begin{aligned} m\angle QCA + w + z &= 180 \\ 90 + 10 + z &= 180 \\ 100 + z &= 180 \\ z &= 80 \end{aligned}$$

m$\angle$BQD = 100 (180 – 80 = 100)
x = 100 (measure of an arc is the same as the measure of its central angle)

9.

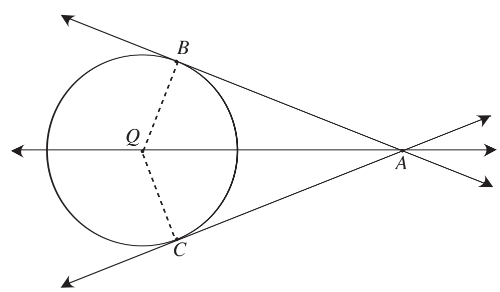
STATEMENT	REASONS
1. $\overline{IJ}$ is a common external tangent of $\odot Q$ and $\odot P$	1. Given
2. $\overline{GH}$ is a common external tangent of $\odot Q$ and $\odot P$	2. Given
3. $\overline{IG} \cong \overline{HG}$	3. Theorem 80 - If two tangent segments are drawn to a circle from the same point, then those segments are congruent.
4. $\overline{HG} \cong \overline{GJ}$	4. Theorem 80
5. $\overline{IG} \cong \overline{GJ}$	5. Transitive Property for Segment Congruence.
6. Point G is the midpoint of $\overline{IJ}$.	6. Definition of Midpoint of a Line Segment

10.

STATEMENT	REASONS
1. $\overline{IJ}$ is a common external tangent of $\odot Q$ and $\odot P$	1. Given
2. $\overline{GH}$ is a common external tangent of $\odot Q$ and $\odot P$	2. Given
3. $\overline{GI} \cong \overline{GH}$	3. Theorem 80 - If two tangent segments are drawn to a circle from the same point, then those segments are congruent.
4. $\overline{GH} \cong \overline{GJ}$	4. Theorem 80
5. $\angle GIH \cong \angle GHI$	5. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
6. $\angle GHJ \cong \angle GJH$	6. Theorem 33
7. $m\angle GIH = m\angle GHI$	7. Definition of Congruent Angles
8. $m\angle GHJ = m\angle GJH$	8. Definition of Congruent Angles
9. $m\angle GIH + m\angle GJH + m\angle IHJ = 180$	9. Theorem 25 - If you have any given triangle, then the sum of the measure of its angles is 180.
10. $m\angle IHJ = m\angle GHI + m\angle GHJ$	10. Postulate 7 - Protractor - Fourth Assumption - Angle Addition Assumption
11. $m\angle GIH + m\angle GJH + m\angle GHI + m\angle GHJ = 180$	11. Substitution (10 into 9)
12. $m\angle GHI + m\angle GHJ + m\angle GHI + m\angle GHJ = 180$	12. Substitution (7 & 8 into 11)
13. $2m\angle GHI + 2m\angle GHJ = 180$	13. Properties of Arithmetic; Collect Like Terms
14. $m\angle GHI + m\angle GHJ = 90$	14. Multiplication Property of Equality
15. $m\angle IHJ = 90$	15. Substitution
16. $\angle IHJ$ is a right angle	16. Definition of Right Angle

11. Corollary 80a - "If two tangent segments are drawn to a circle from the same point, then the line containing that point and the center of the circle bisects the angle formed by the two tangent segments."

Given: $\overline{AB}$ and $\overline{AC}$ are tangent segments to $\odot Q$.
$\overleftrightarrow{AQ}$ contains external point A and point Q.
Prove: $\overleftrightarrow{AQ}$ bisects $\angle BAC$



STATEMENT	REASONS
1. $\overline{AB}$ is a tangent segment to $\odot Q$	1. Given
2. $\overline{AC}$ is a tangent segment to $\odot Q$	2. Given
3. $\overleftrightarrow{AQ}$ contains external point A and point Q	3. Given
4. Draw $\overline{QB}$	4. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
5. Draw $\overline{QC}$	5. Postulate 2
6. $\overline{QB} \cong \overline{QC}$	6. Radii of the same circle are congruent
7. $\overline{AQ} \cong \overline{AQ}$	7. Reflexive Property for Congruent Line Segments
8. $\overline{AB} \cong \overline{AC}$	8. Theorem 80 - If two tangent segments are drawn to a circle from the same point, then those segments are congruent.
9. $\triangle QAP \cong \triangle QAP$	9. Postulate 13 - Triangle Congruence - If the three sides of one triangle are congruent to the three corresponding sides of another triangle, then the two triangles are congruent. (SSS Congruence Assumption)
10. $\angle BAQ \cong \angle CAQ$	10. C.P.C.T.C.
11. $\overleftrightarrow{AQ}$ bisects $\angle BAC$	11. Definition of Angle Bisector

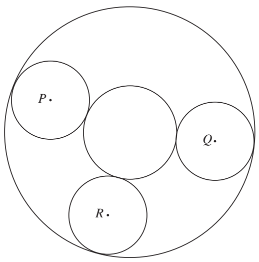
12. Two

13. Zero

14. One

15. No

16. Two



17. **a)** 8 **b)** Infinite number of spheres

18. Conical Surface

19. Cylindrical Surface

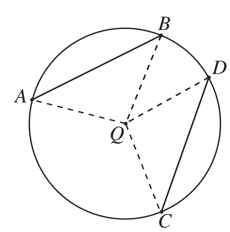
20. QP = QT + TP
In △QRP, QR + RP > QP. (Theorem 40 - If you have the sum of the measures of two sides of a triangle, then that sum is greater than the measure of the third side of the triangle.)
Substituting, we have QR + RP > QT + TP
QR = QT (Radii of the same circle are equal.)
So, RP > TP (Subtraction Property for Inequalities)

Unit VI — Circles

Part C — Line and Segment Relationships

p. 575 – Lesson 7 — Theorem 81 & 82

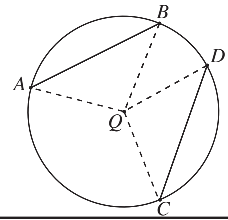
1. Theorem 81 - "If two chords of a circle are congruent, then their minor arcs are congruent."



Given: ⊙Q; $\overline{AB} \cong \overline{CD}$
Prove: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

STATEMENT	REASONS
1. ⊙Q with chord $\overline{AB} \cong \overline{CD}$	1. Given
2. Draw radii $\overline{QA}$, $\overline{QB}$, $\overline{QC}$, and $\overline{QD}$	2. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
3. $\overline{QA} \cong \overline{QB} \cong \overline{QC} \cong \overline{QD}$	3. Radii of the same circle are congruent.
4. △ABQ ≅ △CDQ	4. Postulate 13 - If the three sides of one triangle are congruent to the three corresponding sides of another triangle, then the two triangles are congruent. (SSS Congruence Assumption)
5. ∠AQB ≅ ∠CQD	5. C.P.C.T.C.
6. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$	6. Theorem 65 - In a circle or in congruent circles, if two central angles are congruent, then their corresponding intercepted arcs are congruent.

2. Theorem 82 - "If two minor arcs are congruent, then their chords are congruent."

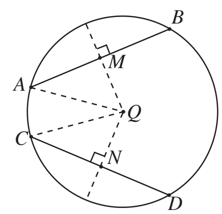


Given: $\odot Q$; $\overset{\frown}{AB} \cong \overset{\frown}{CD}$
Prove: $\overline{AB} \cong \overline{CD}$

STATEMENT	REASONS
1. $\odot Q$ with chord $\overset{\frown}{AB} \cong \overset{\frown}{CD}$	1. Given
2. Draw radii $\overline{QA}$, $\overline{QB}$, $\overline{QC}$, and $\overline{QD}$	2. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
3. Draw chords $\overline{AB}$ and $\overline{CD}$	3. Postulate 2
4. $\overline{QA} \cong \overline{QB} \cong \overline{QC} \cong \overline{QD}$	4. Radii of the same circle are congruent.
5. $\angle AQB \cong \angle CQD$	5. In a circle or in congruent circles, if two minor arcs are congruent, then their corresponding central angles are congruent.
6. $\triangle AQB \cong \triangle CQD$	6. Postulate 13 - Triangle Congruence - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent. (SAS Congruence Assumption)
7. $\overline{AB} \cong \overline{CD}$	7. C.P.C.T.C.

3. **Prove:** In the same circle or congruent circles, two chords are congruent if and only if the two chords are equidistant from the center of the circle.

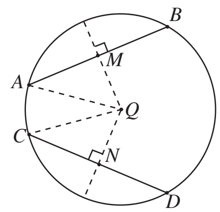
Part 1 - If, in a circle or congruent circles, two chords are congruent, then the two chords are equidistant from the center of the circle.



Given: $\odot Q$; with $\overline{AB} \cong \overline{CD}$
Prove: $\overline{AB}$ and $\overline{CD}$ are equidistant from point Q.

STATEMENT	REASONS
1. $\odot Q$ with $\overline{AB} \cong \overline{CD}$	1. Given
2. Draw radius $\overline{QA}$	2. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
3. Draw radius $\overline{QC}$	3. Postulate 2
4. $\overline{QA} \cong \overline{QC}$	4. Radii of the same circle are congruent.
5. Draw $\overline{QM} \perp \overline{AB}$	5. Postulate 10 - Uniqueness of Perpendicular Lines - In a plane, through a point not on a given line, there is exactly one line (segment) perpendicular to the given line.
6. Draw $\overline{QN} \perp \overline{CD}$	6. Postulate 10 - Uniqueness of Perpendicular Lines
7. $\angle QMA$ is a right angle	7. Definition of Perpendicular Lines (segments)
8. $\angle QNC$ is a right angle	8. Definition of Perpendicular Lines (segments)
9. $\triangle QMA$ is a right triangle	9. Definition of Right Triangle
10. $\triangle QNC$ is a right triangle	10. Definition of Right Triangle
11. $\overline{QM}$ bisects $\overline{AB}$	11. Theorem 73 - If a diameter (radius) of a circle is perpendicular to a chord of that circle, then that diameter (radius) bisects that chord.
12. $\overline{QN}$ bisects $\overline{CD}$	12. Theorem 73
13. $\overline{AM} \cong \overline{BM}$	13. Definition of Line Segment Bisector
14. $\overline{CN} \cong \overline{DN}$	14. Definition of Line Segment Bisector
15. AM = BM	15. Definition of Congruent Line Segments
16. CN = DN	16. Definition of Congruent Line Segments
17. AM + BM = AB	17. Postulate 6 - Ruler - Segment Addition Assumption
18. CN + DN = CD	18. Postulate 6
19. AB = CD	19. Definition of Congruent Line Segments (from step 1)
20. AM + BM = CN + DN	20. Substitution
21. AM + AM = CN + CN	21. Substitution
22. 2AM = 2CN	22. Properties of Arithmetic; Collect Like Terms
23. AM = CN	23. Multiplication Property of Equality
24. $\overline{AM} \cong \overline{CN}$	24. Definition of Congruent Line Segments
25. $\triangle QMA \cong \triangle QNC$	25. Postulate Corollary 13c - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two right triangles are congruent. (HL)
26. $\overline{QM} \cong \overline{QN}$	26. C.P.C.T.C.
27. QM = QN	27. Definition of Congruent Line Segments
28. $\overline{AB}$ and $\overline{CD}$ are equidistant from point Q	28. Definition of Equidistant - Q.E.D.

3. Part 2 - If, in a circle or congruent circles, two chords are equidistant from the center of the circle, then the two chords are congruent.



Given: $\odot Q$ with $\overline{QM} \perp \overline{AB}$ and $\overline{QN} \perp \overline{CD}$
QM = QN
Prove: $\overline{AB} \cong \overline{CD}$

STATEMENT	REASONS
1. $\odot Q$ with $\overline{QM} \perp \overline{AB}$ and $\overline{QN} \perp \overline{CD}$	1. Given
2. QM = QN	2. Given
3. $\overline{QM} \cong \overline{QN}$	3. Definition of Congruent Line Segments
4. Draw $\overline{QA}$	4. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
5. Draw $\overline{QC}$	5. Postulate 2
6. $\overline{QA} \cong \overline{QC}$	6. Radii of the same circle (or congruent circles) are congruent.
7. $\angle$QMA is a right angle	7. Definition of Perpendicular Lines (segments)
8. $\angle$QNC is a right angle	8. Definition of Perpendicular Lines (segments)
9. $\triangle$QMA is a right triangle	9. Definition of Right Triangle
10. $\triangle$QNC is a right triangle	10. Definition of Right Triangle
11. $\triangle QMA \cong \triangle QNC$	11. Postulate Corollary 13c - Hypotenuse Leg Postulate
12. $\overline{AM} \cong \overline{CN}$	12. C.P.C.T.C.
13. $\overline{QM}$ bisects $\overline{AB}$	13. Theorem 73 - If a diameter (radius) of a circle is perpendicular to a chord of that circle, then that diameter (radius) bisects that chord.
14. $\overline{QN}$ bisects $\overline{CD}$	14. Theorem 73
15. $\overline{AM} \cong \overline{BM}$	15. Definition of Line Segment Bisector
16. $\overline{CN} \cong \overline{DN}$	16. Definition of Line Segment Bisector
17. AM = BM	17. Definition of Congruent Line Segments
18. CN = DN	18. Definition of Congruent Line Segments
19. AM + BM = AB	19. Postulate 6 - Ruler - Segment Addition Assumption
20. CN + DN = CD	20. Postulate 6
21. AM = CN	21. Definition of Congruent Line Segments (from step 12)
22. BM = CN	22. Substitution
23. CN + CN = AB	23. Substitution
24. CN + CN = CD	24. Substitution
25. AB = CD	25. Substitution
26. $\overline{AB} \cong \overline{CD}$	26. Definition of Congruent Line Segments. Q.E.D.

Note: the proof of the relationships in this exercise implies that the distance from a given point to a given line is uniquely found by measuring the length of the perpendicular segment from the given point to the given line. It further implies that the distance of the diameter of a circle from its center is zero.

4.

STATEMENT	REASONS
1. $\overline{AB}$ and $\overline{CD}$ are chords of $\odot Q$	1. Given
2. $\overline{AB} \cong \overline{CD}$	2. Given
3. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$	3. Theorem 81 - If two chords of a circle are congruent, then their minor arcs are congruent.
4. $\angle AQB \cong \angle CQD$	4. Theorem 66 - If, in the same circle or congruent circles, two minor arcs are congruent, then the central angles which intercept those arcs are congruent.

5.

$$\begin{aligned} y^2 + 12 &= 2y^2 - 37 \\ 0 &= y^2 - 49 \\ 0 &= (y-7)(y+7) \\ 0 &= y-7 \text{ or } 0 = y+7 \\ y &= 7 \text{ or } y = -7 \\ &\text{(y cannot be negative)} \\ y &= 7 \end{aligned}$$

or

$$\begin{aligned} y^2 + 12 &= 2y^2 - 37 \\ 49 &= y^2 \\ \pm\sqrt{49} &= y \\ &\text{(y cannot be negative)} \\ \sqrt{49} &= y \\ 7 &= y \end{aligned}$$

6. $(5x - 20) + 90 + (5x - 20) + 110 = 360$

$$10x - 40 + 200 = 360$$
$$10x + 160 = 360$$
$$10x = 200$$
$$x = 20$$

7. AB = 12 Since $\overline{AB}$ and $\overline{CD}$ are equidistant from point Q, the center of the circle, $\overline{AB} \cong \overline{CD}$. Therefore, CD = 12

8. *x* has a measure of 76 degrees since the measure of $\widehat{AB}$ is equal to the measure of central angle AQB.
$\overline{AB} \cong \overline{CD}$ so, $m\widehat{AB} = m\widehat{CD}$

$76 = 4y + 20$
$56 = 4y$
$14 = y$

9. $\angle WQV \cong \angle UQV$ because $\overline{QV}$ bisects $\angle UQW$.
$\angle WQV \cong \angle YQX$ by the transitive Property of Angle Congruence ($\angle WQV \cong \angle UQV \cong \angle YQX$)
$\widehat{VW} \cong \widehat{XY}$ (Theorem 65)
Therefore, $\overline{VW} \cong \overline{XY}$ (Theorem 82)

10. The chords of these arcs are congruent by Theorem 82

11.

STATEMENT	REASONS
1. $\odot Q$ with $\overline{XY} \cong \overline{YT}$	1. Given
2. $\overline{QM} \perp \overline{XY}$	2. Given
3. $\overline{QN} \perp \overline{YT}$	3. Given
4. $\overline{QM}$ bisects $\widehat{XY}$	4. Corollary 73a - If a diameter (radius) of a circle is perpendicular to a chord of that circle, then that diameter bisects the arcs intercepted by that chord.
5. $\overline{QN}$ bisects $\widehat{YT}$	5. Corollary 73a
6. $\widehat{XM} \cong \widehat{MY}$	6. Definition of Bisector of an Arc of a Circle.
7. $\widehat{YN} \cong \widehat{NT}$	7. Definition of Bisector of an Arc of a Circle.
8. $m\widehat{XM} = m\widehat{MY}$	8. Definition of Congruent Arcs
9. $m\widehat{YN} = m\widehat{NT}$	9. Definition of Congruent Arcs
10. $m\widehat{XM} + m\widehat{MY} = m\widehat{XY}$	10. Postulate 8 - Circle Arc - Addition Assumption
11. $m\widehat{YN} + m\widehat{NT} = m\widehat{YT}$	11. Postulate 8
12. $\widehat{XY} \cong \widehat{YT}$	12. Theorem 81 - If two chords of a circle, or congruent circles, are congruent, then their minor arcs are congruent.
12. $m\widehat{XY} = m\widehat{YT}$	12. Definition of Congruent Arcs
13. $m\widehat{XM} + m\widehat{MY} = m\widehat{YN} + m\widehat{NT}$	13. Substitution
14. $m\widehat{MY} + m\widehat{MY} = m\widehat{YN} + m\widehat{YN}$	14. Substitution
15. $2m\widehat{MY} = 2m\widehat{YN}$	15. Principles of Arithmetic; Collect Like Terms
16. $m\widehat{MY} = m\widehat{YN}$	16. Multiplication Property of Equality
17. $\widehat{MY} \cong \widehat{YN}$	17. Definition of Congruent Arcs
18. Point Y is the midpoint of $\widehat{MN}$	18. Definition of Midpoint of an Arc. Q.E.D.

12. $m\widehat{AB} + m\widehat{BC} + m\widehat{CD} + 150 = 360$
$m\widehat{AB} = m\widehat{BC} = m\widehat{CD}$

$$m\widehat{BC} + m\widehat{BC} + m\widehat{BC} + 150 = 360$$
$$3m\widehat{BC} = 210$$
$$m\widehat{BC} = 70$$

13. $m\angle VQX = 90$
Therefore, $m\widehat{VX} = 90$

$$m\widehat{VW} + m\widehat{WX} = m\widehat{VX}$$
$$m\widehat{VW} = m\widehat{WX}$$
$$m\widehat{VW} + m\widehat{VW} = 90$$
$$2m\widehat{VW} = 90$$
$$m\widehat{VW} = 45$$

14. $\overline{QV}$ bisects $\overline{UW}$ and $\overline{QF}$ bisects $\overline{EG}$. $\overline{UW}$ and $\overline{EG}$ are the same distance from the center of ⊙Q, so $\overline{UW} \cong \overline{EG}$.

EG = EF + FG	UW = EG
EG = 7 + 7	UW = 14
EG = 14	

15.

STATEMENT	REASONS
1. ⊙Q ≅ ⊙P	1. Given
2. $\overline{CD} \cong \overline{GH}$	2. Given
3. $\overset{\frown}{CD} \cong \overset{\frown}{GH}$	3. Theorem 81 - If two chords of a circle or congruent circles are congruent, then their minor arcs are congruent.

16.

STATEMENT	REASONS
1. ⊙Q ≅ ⊙P	1. Given
2. $\overline{QX} \cong \overline{PY}$	2. Given
3. QX = PY	3. Definition of Congruent Line Segments
4. $\overline{CD} \cong \overline{GH}$	4. Exercise 3 - In the same circle or in congruent circles, two chords are congruent, if and only if, the two chords are equidistant from the center of the circle.

17.

STATEMENT	REASONS
1. ⊙Q ≅ ⊙P	1. Given
2. $\overline{QX} \cong \overline{PY}$	2. Given
3. QX = PY	3. Definition of Congruent Line Segments
4. $\overline{CD} \cong \overline{GH}$	4. Exercise 3 - In the same circle or in congruent circles, two chords are congruent, if and only if, the two chords are equidistant from the center of the circle.
5. $\overset{\frown}{CD} \cong \overset{\frown}{GH}$	5. Theorem 81 - If two chords of a circle or congruent circles are congruent, then their minor arcs are congruent. Q.E.D.

18.

STATEMENT	REASONS
1. $\overline{UV}$ is a diameter of ⊙Q	1. Given
2. $\overline{UV} \perp \overline{WY}$	2. Given
3. $\overset{\frown}{WV} \cong \overset{\frown}{YV}$	3. Corollary 73a - If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects the arcs intercepted by that chord.

19.

STATEMENT	REASONS
1. $\overline{UV}$ is a diameter of ⊙Q	1. Given
2. $\overline{UV} \perp \overline{WY}$	2. Given
3. $\overline{XW} \cong \overline{XY}$	3. Theorem 73 - If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects that chord.
4. $\overset{\frown}{WV} \cong \overset{\frown}{YV}$	4. Proven in exercise 18
5. $\overline{WV} \cong \overline{YV}$	5. Theorem 82 - If two minor arcs of a circle, or congruent circles, are congruent, then their chords are congruent.
6. $\overline{VX} \cong \overline{VX}$	6. Reflexive Property for Congruent Line Segments
7. △VXW ≅ △VXY	7. Postulate 13 - Triangle Congruence - If the three sides of one triangle are congruent to the three corresponding sides of another triangle, then the two triangles are congruent. (SSS Congruence Assumption)

20. a)

$$(MR)^2 + (MQ)^2 = (QR)^2$$
$$(MR)^2 + (8)^2 = (10)^2$$
$$(MR)^2 + 64 = 100$$
$$(MR)^2 = 36$$
$$MR = \pm\sqrt{36} \text{ (MR cannot be negative)}$$
$$MR = 6$$

$$\overline{PM} \cong \overline{MR}$$
$$PM + MR = PR$$
$$6 + 6 = PR$$
$$12 = PR$$

$$(NT)^2 + (NQ)^2 = (QT)^2$$
$$(NT)^2 + (5)^2 = (10)^2$$
$$(NT)^2 + 25 = 100$$
$$(NT)^2 = 75$$
$$NT = \pm\sqrt{75} \text{ (NT cannot be negative)}$$
$$NT = \sqrt{75}$$

$$\overline{UN} \cong \overline{NT}$$
$$UN + NT = UT$$
$$\sqrt{75} + \sqrt{75} = UT$$
$$2\sqrt{75} = UT$$
$$2\sqrt{25 \cdot 3} = UT$$
$$2 \cdot 5 \cdot \sqrt{3} = UT$$
$$10\sqrt{3} = UT$$
$$17.32 \approx UT$$

b) Since $10\sqrt{3} > 12$, $UT > PR$.
Since QM > QN, $\overline{PR}$ is farther from point Q than is $\overline{UT}$.

c) In the same circle or congruent circles, if two chords are not congruent, then their distances from the center of the circle are not equal, and the longer chord is closer to the center of the circle. The converse is also true.

21. a) False
b) True
c) True
d) False
e) True
f) False
g) True
h) False
i) False
j) False

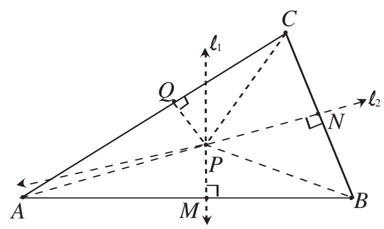
Unit VI — Circles

Part D — Circles Concurrency

p. 575 – Lesson 1 — Theorem 83

1.

Given: $\triangle ABC$; $\overline{AB}$ is a chord of some circle
Prove: $\triangle ABC$ is cyclic



STATEMENT	REASONS
1. $\triangle ABC$	1. Given
2. $\overline{AB}$ is a chord of some circle.	2. Given
3. Locate point M on $\overline{AB}$ as the midpoint of $\overline{AB}$.	3. Theorem 4 - If you have a given line segment, then that segment has exactly one midpoint.
4. Draw ℓ_1 perpendicular to $\overline{AB}$ at point M.	4. Theorem 6 - If, in a plane, there is a point on a line, then there is exactly one perpendicular to the line through that point.
5. ℓ_1 contains a diameter of the circle which has $\overline{AB}$ as a chord.	5. Theorem 75 - If a chord of a circle (ℓ_1) is a perpendicular bisector of another chord ($\overline{AB}$) of that circle, then the original chord (ℓ_1) must be a diameter of the circle.
6. ℓ_1 must pass through the center of the circle.	6. Definition of Diameter
7. Locate point N on $\overline{BC}$ as the midpoint of $\overline{BC}$.	7. Theorem 4
8. Draw ℓ_2 perpendicular to $\overline{BC}$ at point N.	8. Theorem 6
9. Call the intersection of ℓ_1 and ℓ_2 point P.	9. Postulate 5 - If two different lines intersect, the intersection is a unique point.
10. Draw $\overline{PA}$.	10. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
11. Draw $\overline{PB}$.	11. Postulate 2
12. Draw $\overline{PC}$.	12. Postulate 2
13. $\angle PMB$ is a right angle.	13. Definition of Perpendicular Lines
14. $\triangle PMB$ is a right triangle.	14. Definition of Right Triangle
15. $\angle PMA$ is a right angle.	15. Definition of Perpendicular Lines
16. $\triangle PMA$ is a right triangle.	16. Definition of Right Triangle
17. $\overline{MA} \cong \overline{MB}$	17. Definition of Midpoint of a Line Segment
18. $\overline{PM} \cong \overline{PM}$	18. Reflexive Property for Congruent Line Segments
19. $\triangle PMA \cong \triangle PMB$	19. Postulate Corollary 13b - If the two legs of a right triangle are congruent to the two legs of another right triangle, then the two right triangles are congruent. (LL)
20. $\overline{PA} \cong \overline{PB}$	20. C.P.C.T.C.
21. $\angle PNB$ is a right angle.	21. Definition of Perpendicular Lines
22. $\triangle PNB$ is a right triangle.	22. Definition of Right Triangle
23. $\overline{NC} \cong \overline{NB}$	23. Definition of Midpoint of a Line Segment
24. $\overline{PN} \cong \overline{PN}$	24. Reflexive Property for Congruent Line Segments
25. $\triangle PNB = \triangle PNC$	25. Postulate Corollary 13b
26. $\overline{PB} \cong \overline{PC}$	26. C.P.C.T.C.
27. $PA = PB$	27. Definition of Congruent Line Segments
28. $PB = PC$	28. Definition of Congruent Line Segments
29. $PA = PB = PC$	29. Transitive Property of Equality
30. $\triangle ABC$ is cyclic.	30. Using $\overline{PA}$, $\overline{PB}$ and $\overline{PC}$ as radii, draw a circle with point P as the center passing through points A, B and C.

2. $\triangle ABC$ is cyclic. (Theorem 83)
$\overline{AC}$ is a diameter.
$m\widehat{AC} = 180$

$m\angle ABC = \frac{1}{2} \cdot m\widehat{AC}$
$= \frac{1}{2} \cdot 180$
$= 90$ (Theorem 67)

$m\angle BAC = \frac{1}{2} \cdot m\widehat{BC}$
$55 = \frac{1}{2} \cdot m\widehat{BC}$
$110 = m\widehat{BC}$ (Theorem 67)

$m\angle ACB = \frac{1}{2} \cdot m\widehat{AB}$
$= \frac{1}{2} \cdot 70$
$= 35$

$m\angle ABC = x^{\circ}$
$= 90^{\circ}$

$m\widehat{AB} = 180 - m\widehat{BC}$
$= 180 - 110$
$= 70$

$m\angle ACB = y^{\circ}$
$= 35^{\circ}$

3. $\triangle ACB$ is cyclic.

$m\widehat{AB} + m\widehat{BC} + m\widehat{CA} = 360^{\circ}$
$6y^{\circ} + 4x^{\circ} + 6y^{\circ} = 360^{\circ}$
$6y^{\circ} + 6y^{\circ} + 6y^{\circ} = 360^{\circ}$
$18y^{\circ} = 360^{\circ}$
$y^{\circ} = 20^{\circ}$

$m\angle ACB = \frac{1}{2} \cdot 6y$
$2x^{\circ} = \frac{1}{2} \cdot 6y$
$4x^{\circ} = 6y^{\circ}$
$4x^{\circ} = (6)(20)$
$4x^{\circ} = 120$
$x^{\circ} = 30$

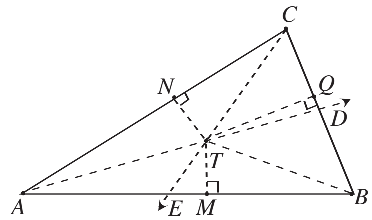
4. Prove Corollary 83a - "If you have a triangle, then the perpendicular bisectors of the sides are concurrent."
(Refer to the figure in Exercise 1)

STATEMENT	REASONS
1. Perpendicular bisectors $\overline{PM}$ and $\overline{PN}$ cross at point P, the center of the circle circumscribing $\triangle ABC$.	1. Proved in Exercise 1
2. Locate point Q on $\overline{AC}$ as the midpoint of $\overline{AC}$.	2. Theorem 4 - If you have a given line segment, then that segment has exactly one midpoint.
3. Draw $\overline{PQ}$.	3. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
4. $\overline{QA} \cong \overline{QC}$	4. Definition of Midpoint
5. $\overline{PQ}$ bisects $\overline{AC}$.	5. Definition of Bisector of a Line Segment
6. $\overline{PQ}$ is a diameter of Circle Q.	6. Definition of Diameter - A segment containing the center of the circle
7. $\overline{PQ} \perp \overline{AC}$	7. Theorem 74 - If a diameter of a circle bisects a chord of the circle, then that diameter is perpendicular to that chord.
8. $\overline{PQ}$, $\overline{PM}$ and $\overline{PN}$ (all three perpendicular bisectors of the sides) pass through the center of the circle P.	8. Q.E.D.

5.

Given: $\triangle ABC$;

Prove: Angle Bisectors of $\angle A$, $\angle B$, and $\angle C$ are concurrent.



STATEMENT	REASONS
1. $\triangle ABC$	1. Given
2. Draw $\overrightarrow{AD}$, the bisector of $\angle CAB$.	2. Theorem 8 - If, in a half plane, there is a given angle, then that angle has exactly one bisector.
3. Draw $\overrightarrow{CE}$, the bisector of $\angle ACB$.	3. Theorem 8
4. Call the intersection of $\overrightarrow{AD}$ and $\overrightarrow{CE}$ point T.	4. Postulate 5 - If two different lines (rays) intersect, the intersection is a unique point.
5. Draw $\overline{TM}$ perpendicular to $\overline{AB}$ at point M.	5. Postulate 10 - In a plane, through a point not on a given line, there is exactly one line perpendicular to the given line.
6. Draw $\overline{TN}$ perpendicular to $\overline{AC}$ at point N.	6. Postulate 10
7. Draw $\overline{TQ}$ perpendicular to $\overline{BC}$ at point Q.	7. Postulate 10
8. $\angle AMT$ is a right angle.	8. Definition of Perpendicular Lines
9. $\triangle AMT$ is a right triangle.	9. Definition of Right Triangle
10. $\angle ANT$ is a right angle.	10. Defintion of Perpendicular Lines
11. $\triangle ANT$ is a right triangle.	11. Definition of Right Triangle
12. $\angle NAT \cong \angle MAT$	12. Definition of Angle Bisector
13. $\overline{AT} \cong \overline{AT}$	13. Reflexive Property of Congruent Line Segments
14. $\triangle NAT \cong \triangle MAT$	14. Postulate Corollary 13a - If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the two right triangles are congruent. (HA)
15. $\overline{NT} \cong \overline{MT}$	15. C.P.C.T.C.
16. $\angle CQT$ is a right angle.	16. Definition of Perpendicular Lines
17. $\triangle CQT$ is a right triangle.	17. Definition of Right Triangle
18. $\angle CNT$ is a right angle.	18. Definition of Perpendicular Lines
19. $\triangle CNT$ is a right triangle.	19. Definition of Right Triangle
20. $\angle NCT \cong \angle QCT$	20. Definition of Angle Bisector
21. $\overline{CT} \cong \overline{CT}$	21. Reflexive Property of Congruent Line Segments
22. $\triangle NCT \cong \triangle QCT$	22. Postulate Corollary 13a (HA)
23. $\overline{NT} \cong \overline{QT}$	23. C.P.C.T.C.
24. NT = MT	24. Definition of Congruent Line Segments
25. NT = QT	25. Definition of Congruent Line Segments
26. NT = MT = QT	26. Transititive Property of Equality
27. Draw $\overline{BT}$	27. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
28. $\angle BMT$ is a right angle.	28. Definition of Perpendicular Lines
29. $\triangle BMT$ is a right triangle.	29. Definition of Right Triangle
30. $\angle BQT$ is a right angle.	30. Definition of Perpendicular Lines
31. $\triangle BQT$ is a right triangle.	31. Definition of Right Triangle
32. $\overline{MT} \cong \overline{QT}$	32. Transitive Property for Congruent Line Segments
33. $\overline{BT} \cong \overline{BT}$	33. Reflexive Property of Congruent Line Segments
34. $\triangle BMT \cong \triangle BQT$	34. Postulate Corollary 13c - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two right triangles are congruent. (HL)
35. $\angle DBT \cong \angle MBT$	35. C.P.C.T.C.
36. $\overline{BT}$ bisects $\angle MBQ$.	36. Definition of Angle Bisector
37. The angle bisectors $\overline{AT}$, $\overline{CT}$ and $\overline{BT}$ pass through point T.	37. Q.E.D.

6. **a)** bisect
b) Yes

7. **a)** Perpendicular Bisectors
b) Yes

8. **a)** The midpoint of a line segment is given by $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ where P_1 (x_1, y_1) and P_2 (x_2, y_2) are the endpoints of the line segment.

Midpoint of $\overline{AB}$ is $R\left(\frac{2a+0}{2},\frac{0+2b}{2}\right)$, $R\left(\frac{2a}{2},\frac{2b}{2}\right)$, $R(a,b)$

Midpoint of $\overline{BC}$ is $S\left(\frac{0+2c}{2},\frac{2b+0}{2}\right)$, $S\left(\frac{2c}{2},\frac{2b}{2}\right)$, $S(c,b)$

Midpoint of $\overline{AC}$ is $T\left(\frac{2a+2c}{2},\frac{0+0}{2}\right)$, $T\left(\frac{\cancel{2}(a+c)}{\cancel{2}},\frac{0}{2}\right)$, $T(a+c,0)$

b) The slope of a line is given by $m=\frac{\text{Rise}}{\text{Run}}=\frac{y_2-y_1}{x_2-x_1}$, where (x_1, y_1) and (x_2, y_2) are coordinates of two points on the line.

Slope of $\overline{AB}$ is
$$m=\frac{2b-0}{0-2a}$$
$$m=\frac{2b}{-2a}$$
$$m=\frac{\cancel{2}b}{-1\cdot\cancel{2}a}$$
$$m=-\frac{b}{a}$$

Slope of $\overline{BC}$ is
$$m=\frac{0-2b}{2c-0}$$
$$m=\frac{-2b}{2c}$$
$$m=\frac{-1\cdot\cancel{2}\cdot b}{\cancel{2}c}$$
$$m=-\frac{b}{c}$$

Slope of $\overline{AC}$ is
$$m=\frac{0-0}{2c-2a}$$
$$m=\frac{0}{2c-2a} \qquad c\neq 0, a\neq 0$$
$$m=0 \qquad (\overline{AC}\text{ is horizontal})$$

The equation of the line perpendicular to $\overline{AB}$ and passing through point R (a, b): Slope of perpendicular line is the opposite reciprocal of the slope of $\overline{AB}$. The opposite reciprocal of $-\frac{b}{a}$ is $+\frac{a}{b}$. Use the point-slope form of the equation of a line.

$$y-y_1=m(x-x_1)$$
$$y-b=\frac{a}{b}(x-a)$$
$$y=\frac{a}{b}(x-a)+b \qquad \text{(equation 1)}$$

The equation of the line perpendicular to $\overline{BC}$ and passing through point S (c, b): Slope of perpendicular line is the opposite reciprocal of the slope of $\overline{BC}$. The opposite reciprocal of $-\frac{b}{c}$ is $+\frac{c}{b}$. Use the point-slope form of the equation of a line.

$$y-y_1=m(x-x_1)$$
$$y-b=\frac{c}{b}(x-c)$$
$$y=\frac{c}{b}(x-c)+b \qquad \text{(equation 2)}$$

The equation of the line perpendicular to $\overline{AC}$ and passing through point T (a + c, 0): Slope of $\overline{AC}$ is 0. This means $\overline{AC}$ is a horizontal line. The perpendicular to $\overline{AC}$ will, therefore have an undefined slope and is a vertical line. The equation is *x = a + c.*

$$x=a+c \qquad \text{(equation 3)}$$

c) Solve simultaneously equation 1 and 2 by the method of elimination by addition to find x.

1) $y = \frac{a}{b}(x-a)+b$

$y = \frac{ax}{b} - \frac{a^2}{b} + b$

2) $y = \frac{c}{b}(x-c)+b$

$y = \frac{cx}{b} - \frac{c^2}{b} + b$

$$y = \frac{ax}{b} - \frac{a^2}{b} + b$$
$$-y = -\frac{cx}{b} + \frac{c^2}{b} - b$$
$$0 = \frac{ax}{b} - \frac{cx}{b} - \frac{a^2}{b} + \frac{c^2}{b}$$
$$0 = x\left(\frac{a}{b} - \frac{c}{b}\right) + \frac{c^2-a^2}{b}$$
$$\frac{a^2-c^2}{b} = x\left(\frac{a-c}{b}\right)$$
$$\left(\frac{b}{a-c}\right)\left(\frac{a^2-c^2}{b}\right) = x\left(\frac{a-c}{b}\right)\left(\frac{b}{a-c}\right)$$
$$\frac{\cancel{(b)}\cancel{(a-c)}(a+c)}{\cancel{(a-c)}\cancel{(b)}} = x \cdot 1$$
$$a+c = x$$

Solve simultaneously equation 1 and 2 by the method of elimination by addition to find y.

1) $y-b = \frac{c}{b}(x-c)$

$y-b = \frac{cx}{b} - \frac{c^2}{b}$

$y-b+\frac{c^2}{b} = \frac{cx}{b}$

$\frac{b}{c}\cdot\left[y-b+\frac{c^2}{b}\right] = \frac{b}{c}\cdot\frac{cx}{b}$

$\frac{by}{c} - \frac{b^2}{c} + \frac{\cancel{b}c^2}{c\cancel{b}_1} = x$

$\frac{by}{c} - \frac{b^2}{c} + \frac{c^2}{c} = x$

2) $y-b = \frac{a}{b}(x-a)$

$y-b = \frac{ax}{b} - \frac{a^2}{b}$

$y-b+\frac{a^2}{b} = \frac{ax}{b}$

$\frac{b}{a}\cdot\left[y-b+\frac{a^2}{b}\right] = \frac{\cancel{b}}{\cancel{a}}\cdot\frac{\cancel{a}x}{\cancel{b}}$

$\frac{by}{a} - \frac{b^2}{a} + \frac{\cancel{b}a^2}{a\cancel{b}_1} = x$

$\frac{by}{a} - \frac{b^2}{a} + \frac{a^2}{a} = x$

Since both equations equal x, set them equal to each other and solve for y.

$$\frac{by}{c} - \frac{b^2}{c} + \frac{c^2}{c} = \frac{by}{a} - \frac{b^2}{a} + \frac{a^2}{a}$$
$$\frac{by}{c} - \frac{by}{a} = \frac{b^2}{c} - \frac{c^2}{c} - \frac{b^2}{a} + \frac{a^2}{a}$$
$$y\left(\frac{b}{c} - \frac{b}{a}\right) = \frac{b^2-c^2}{c} + \frac{a^2-b^2}{a}$$
$$y\left(\frac{b}{c} - \frac{b}{a}\right)\left(\frac{ac}{b}\right) = \left(\frac{b^2-c^2}{c} + \frac{a^2-b^2}{a}\right)\left(\frac{ac}{b}\right)$$
$$y\left(\frac{\cancel{b}a\cancel{c}}{\cancel{c}\cancel{b}} - \frac{\cancel{b}\cancel{a}c}{\cancel{a}\cancel{b}}\right) = \frac{(b^2-c^2)}{\cancel{c}}\cdot\frac{a\cancel{c}}{b} + \frac{(a^2-b^2)}{\cancel{a}}\cdot\frac{\cancel{a}c}{b}$$
$$y(a-c) = \frac{ab^2-ac^2}{b} + \frac{a^2c-b^2c}{b}$$
$$y(a-c) = \frac{ab^2-ac^2+a^2c-b^2c}{b}$$
$$y(a-c) = \frac{ab^2-b^2c+a^2c-ac^2}{b}$$
$$y(a-c) = \frac{b^2(a-c)+ac(a-c)}{b}$$
$$y(a-c) = \frac{(b^2+ac)(a-c)}{b}$$
$$y\cancel{(a-c)}\cdot\frac{1}{\cancel{(a-c)}} = \frac{(b^2+ac)\cancel{(a-c)}}{b}\cdot\frac{1}{\cancel{(a-c)}}$$
$$y = \frac{(b^2+ac)}{b}$$

Equations 1 and 2 intersect at $\left(a+c, \frac{b^2+ac}{b}\right)$

Solve simultaneously equation 2 and 3 by the substitution method.

2) $y-b=\frac{c}{b}(x-c)$ 3) $x = a + c$

$$y-b=\frac{c}{b}(a+c-c)$$

$$y-b=\frac{c}{b}(a)$$

$$y-b=\frac{ac}{b}$$

$$y=\frac{ac}{b}+b$$

$$y=\frac{ac}{b}+\frac{b^2}{b}$$

$$y=\frac{ac+b^2}{b}$$

Equations 2 and 3 intersect at $\left(a+c, \frac{b^2+ac}{b}\right)$

Solve simultaneously equation 1 and 3 by the substitution method.

1) $y-b=\frac{a}{b}(x-a)$ 3) $x = a + c$

$$y-b=\frac{a}{b}(a+c-a)$$

$$y-b=\frac{a}{b}(c)$$

$$y-b=\frac{ac}{b}$$

$$y=\frac{ac}{b}+b$$

$$y=\frac{ac}{b}+\frac{b^2}{b}$$

$$y=\frac{ac+b^2}{b}$$

Equations 1 and 3 intersect at $\left(a+c, \frac{b^2+ac}{b}\right)$

d) Distance Formula: $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Point I $\left(a+c, \frac{b^2+ac}{b}\right)$ to Point A (2a, 0)

$$D=\sqrt{(2a-(a+c))^2+\left(0-\frac{b^2+ac}{b}\right)^2}$$

$$=\sqrt{(2a-a-c)^2+\left(-\frac{b^2+ac}{b}\right)^2}$$

$$=\sqrt{(a-c)^2+\left(\frac{-b^2-ac}{b}\right)^2}$$

$$=\sqrt{(a-c)(a-c)+\left(\frac{-b^2-ac}{b}\right)\left(\frac{-b^2-ac}{b}\right)}$$

$$=\sqrt{a^2-2ac+c^2+\frac{b^4+2b^2ac+a^2c^2}{b^2}}$$

$$=\sqrt{\frac{a^2b^2-2b^2ac+b^2c^2}{b^2}+\frac{b^4+2b^2ac+a^2c^2}{b^2}}$$

$$=\sqrt{\frac{b^4+a^2b^2+a^2c^2+b^2c^2}{b^2}}$$

$$=\sqrt{\frac{b^4}{b^2}+\frac{a^2b^2}{b^2}+\frac{a^2c^2}{b^2}+\frac{b^2c^2}{b^2}}$$

$$=\sqrt{a^2+b^2+c^2+\frac{a^2c^2}{b^2}}$$

Point I $\left(a+c, \frac{b^2+ac}{b}\right)$ to Point B (0, 2b)

$$D=\sqrt{(0-(a+c))^2+\left(2b-\frac{b^2+ac}{b}\right)^2}$$

$$=\sqrt{(-a-c)^2+\left(2b-\frac{b^2+ac}{b}\right)^2}$$

$$=\sqrt{(-a-c)^2+\left(\frac{2b^2}{b}-\frac{b^2+ac}{b}\right)^2}$$

$$=\sqrt{(-a-c)^2+\left(\frac{2b^2-b^2-ac}{b}\right)^2}$$

$$=\sqrt{(-a-c)^2+\left(\frac{b^2-ac}{b}\right)^2}$$

$$=\sqrt{(-a-c)(-a-c)+\left(\frac{b^2-ac}{b}\right)\left(\frac{b^2-ac}{b}\right)}$$

$$=\sqrt{a^2+2ac+c^2+\frac{b^4-2b^2ac+a^2c^2}{b^2}}$$

$$=\sqrt{\frac{a^2b^2+2b^2ac+b^2c^2}{b^2}+\frac{b^4-2b^2ac+a^2c^2}{b^2}}$$

$$=\sqrt{\frac{b^4+a^2b^2+a^2c^2+b^2c^2}{b^2}}$$

$$=\sqrt{\frac{b^4}{b^2}+\frac{a^2b^2}{b^2}+\frac{a^2c^2}{b^2}+\frac{b^2c^2}{b^2}}$$

$$=\sqrt{a^2+b^2+c^2+\frac{a^2c^2}{b^2}}$$

Point I $\left(a+c, \frac{b^2+ac}{b}\right)$ to Point C (2c, 0)

$$D = \sqrt{\left(2c-(a+c)\right)^2 + \left(0-\frac{b^2+ac}{b}\right)^2}$$
$$= \sqrt{(2c-a-c)^2 + \left(-\frac{b^2+ac}{b}\right)^2}$$
$$= \sqrt{(c-a)^2 + \left(\frac{-b^2-ac}{b}\right)^2}$$
$$= \sqrt{(c-a)(c-a) + \left(\frac{-b^2-ac}{b}\right)\left(\frac{-b^2-ac}{b}\right)}$$
$$= \sqrt{c^2 - 2ac + a^2 + \frac{b^4+2b^2ac+a^2c^2}{b^2}}$$
$$= \sqrt{\frac{b^2c^2-2b^2ac+a^2b^2}{b^2} + \frac{b^4+2b^2ac+a^2c^2}{b^2}}$$
$$= \sqrt{\frac{b^4+a^2b^2+a^2c^2+b^2c^2}{b^2}}$$
$$= \sqrt{\frac{b^4}{b^2}+\frac{a^2b^2}{b^2}+\frac{a^2c^2}{b^2}+\frac{b^2c^2}{b^2}}$$
$$= \sqrt{a^2+b^2+c^2+\frac{a^2c^2}{b^2}}$$

Summary: The perpendicular bisects meet at point I $\left(a+c, \frac{b^2+ac}{b}\right)$.

The distance from each side of ABC to point I is $\sqrt{a^2+b^2+c^2+\frac{a^2c^2}{b^2}}$.

9. $\overline{QN}$ bisects $\overline{BC}$.

$$BN = \frac{1}{2}BC$$
$$= \frac{1}{2}\cdot 48$$
$$= 24$$

In $\triangle BNQ$, $(BN)^2 + (NQ)^2 = (BQ)^2$

$$(24)^2 + (7)^2 = (BQ)^2$$
$$576 + 49 = (BQ)^2$$
$$625 = (BQ)^2$$
$$\pm\sqrt{625} = BQ \quad \text{(BQ cannot be negative)}$$
$$25 = BQ$$
$$QA = BQ \quad \text{(Corollary 83a)}$$
$$QA = 25$$

10. In $\triangle XPQ$, $(XP)^2 + (PQ)^2 = (XQ)^2$

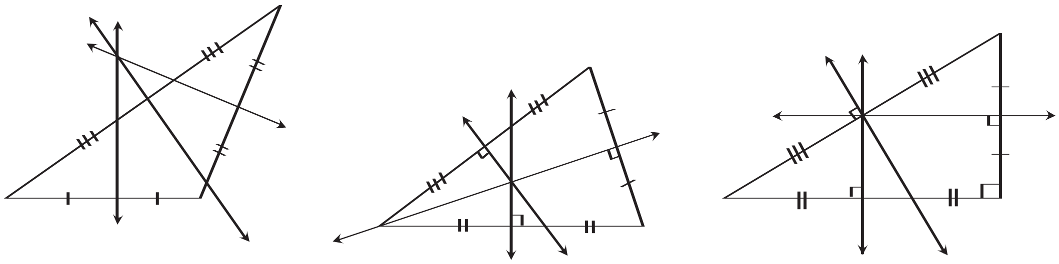
$$(4)^2 + (PQ)^2 = (5)^2$$
$$16 + (PQ)^2 = 25$$
$$(PQ)^2 = 9$$
$$PQ = \pm\sqrt{9} \quad \text{(PQ cannot be negative)}$$
$$PQ = 3$$
$$PQ = QM \quad \text{(Corollary 83b)}$$
$$3 = QM$$

11. RQ = SQ (Corollary 83a)
RQ = 11

12. The perpendicular bisectors of the sides of a triangle intersect in a point that is equidistant from the vertices of the triangle. Point Q is not necessarily equidistant from the sides of the triangle. Point Q is equidistant from point A, point B, and point C. (Corollary 83a)

13. The angle bisectors of a triangle intersect in a point that is equidistant from the sides of the triangle, but QZ and QX are not necessarily distances to the sides. Point Q is equidistant from $\overline{MN}$, $\overline{NP}$ and $\overline{PM}$. (Corollary 83b)

14.



15. MB = MA (Corollary 83a)

In $\triangle APM$, $(AP)^2 + (MP)^2 = (MA)^2$

$$(AP)^2 + (12)^2 = (13)^2$$

$$(AP)^2 + 144 = 169$$

$$(AP)^2 = 25$$

$$AP = \pm\sqrt{25} \quad \text{(AP cannot be negative)}$$

$$AP = 5$$

Since $\overline{MP}$ bisects $\overline{AC}$ and AC = AP + PC, then AC = 5 + 5 = 10

16. Always
17. Always
18. Never
19. Sometimes
20. Always

21. 180 – 34 = 146

$$m\angle YXZ + m\angle YZX = 68$$

$$\frac{1}{2}(m\angle YXZ + m\angle YZX) = 34$$

$$m\angle PXZ + m\angle PZX = 34$$

22. The midpoint of a line segment is given by: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

The midpoint of $\overline{AB}$: $\left(\frac{0+16}{2}, \frac{0+0}{2}\right)$, $\left(\frac{16}{2}, \frac{0}{2}\right)$, $(8,0)$

The midpoint of $\overline{BC}$: $\left(\frac{16+12}{2}, \frac{0+8}{2}\right)$, $\left(\frac{28}{2}, \frac{8}{2}\right)$, $(14,4)$

The midpoint of $\overline{AC}$: $\left(\frac{0+12}{2}, \frac{0+8}{2}\right)$, $\left(\frac{12}{2}, \frac{8}{2}\right)$, $(6,4)$

Slope of $\overline{AB}$: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-0}{16-0} = \frac{0}{16} = 0$

Slope of line perpendicular to $\overline{AB}$: Undefined

Slope of $\overline{BC}$: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{12-16} = \frac{8}{-4} = -2$

Slope of line perpendicular to $\overline{BC}$: $\frac{1}{2}$

Slope of $\overline{AC}$: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{12-0} = \frac{8}{12} = \frac{\cancel{4} \cdot 2}{\cancel{4} \cdot 3} = \frac{2}{3}$

Slope of line perpendicular to $\overline{AC}$: $-\frac{3}{2}$

22. continued

Equation of line perpendicular to $\overline{AB}$:
The line is vertical passing through (8, 0)

$$x = 8$$

Equation of line perpendicular to $\overline{BC}$:

$$y - y_1 = m(x - x_1)$$
$$m = \frac{1}{2} \text{ and } (x_1, y_1) = (14, 4)$$
$$y - 4 = \frac{1}{2}(x - 14)$$
$$y - 4 = \frac{1}{2}x - 7$$
$$y = \frac{1}{2}x - 3$$

Equation of line perpendicular to $\overline{AC}$:

$$y - y_1 = m(x - x_1)$$
$$m = -\frac{3}{2} \text{ and } (x_1, y_1) = (6, 4)$$
$$y - 4 = -\frac{3}{2}(x - 6)$$
$$y - 4 = -\frac{3}{2}x + 9$$
$$y = -\frac{3}{2}x + 13$$

23. Intersection of 1 and 2:

1) $x = 8$

2) $y = \frac{1}{2}x - 3$

$= \frac{1}{2}8 - 3$

$= 4 - 3$

$= 1$

Intersection of 1 and 3:

1) $x = 8$

3) $y = -\frac{3}{2}x + 13$

$= -\frac{3}{2}(8) + 13$

$= -12 + 13$

$= 1$

Intersection of 2 and 3:

2) $y = \frac{1}{2}x - 3$

3) $y = -\frac{3}{2}x + 13$

Use substitution to find x and y.

$$\frac{1}{2}x - 3 = -\frac{3}{2}x + 13$$

$\frac{4}{2}x = 16$

$2x = 16$

$x = 8$

$y = \frac{1}{2}x - 3$

$= \frac{1}{2}(8) - 3$

$= 4 - 3$

$= 1$

Intersection point is (8, 1)

24. Distance from point A (0,0) to (8,1):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(8 - 0)^2 + (1 - 0)^2}$$
$$= \sqrt{(8)^2 + (1)^2}$$
$$= \sqrt{64 + 1}$$
$$= \sqrt{65}$$

Distance from point B (16,0) to (8,1):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(8 - 16)^2 + (1 - 0)^2}$$
$$= \sqrt{(-8)^2 + (1)^2}$$
$$= \sqrt{64 + 1}$$
$$= \sqrt{65}$$

Distance from point C (12,8) to (8,1):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(8 - 12)^2 + (1 - 8)^2}$$
$$= \sqrt{(-4)^2 + (-7)^2}$$
$$= \sqrt{16 + 49}$$
$$= \sqrt{65}$$

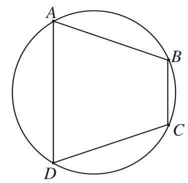
Unit VI — Circles

Part D — Circles Concurrency

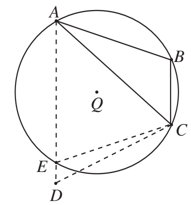
p. 575 – Lesson 2 — Theorem 84

1. Theorem 84 - "If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic."

Given: Quadrilateral ABCD
∠DAB and ∠BCD are supplementary
∠ABC and ∠ADC are supplementary
Prove: Quadrilateral ABCD is cyclic

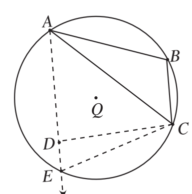


Case 1: Consider three non-collinear points A, B, and C as a triangle. Point D is outside ⊙Q.



STATEMENT	REASONS
1. Quadrilateral ABCD	1. Given
2. ∠A and ∠C are supplementary.	2. Given
3. ∠B and ∠D are supplementary.	3. Given
4. Non-collinear points A, B, and C form a triangle.	4. Definition of a Triangle - A polygon formed by three non-collinear segments connected at their endpoints.
5. There exists a circle which passes through the vertices of ABC, call it ⊙Q.	5. Theorem 83 - If you have a triangle, then that triangle is cyclic
6. Draw $\overline{CE}$, from point C to point E, where ⊙Q intersects side $\overline{AD}$ of quadrilateral ABCD.	6. Postulate 2 - For any two points there is exactly one line (segment) containing them.
7. ∠ABC is supplementary to ∠AEC.	7. Corollary 67b - If you have a quadrilateral inscribed in a circle, then its opposite angles are supplementary.
8. ∠ADC ≅ ∠AEC	8. Theorem 14 - If two angles are supplementary to the same angle or congruent angles, then they are congruent to each other.
9. However, ∠ADC cannot be congruent to ∠AEC since m∠AEC > m∠ADC.	9. Theorem 37 - If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angles.
10. Point D cannot be outside ⊙Q since, if it was, ∠ADC would not be supplementary to ∠ABC.	10. Contradiction of given that ∠ABC and ∠ADC are supplementary angles.

Case 2: Consider three non-collinear points A, B, and C as a triangle. Point D is inside ⊙Q. Extend side $\overline{AD}$ to meet circle Q at point E.



STATEMENT	REASONS
11. Draw $\overline{EC}$	11. Postulate 2
12. ∠ABC is supplementary to ∠AEC.	12. Corollary 67b
13. ∠ADC ≅ ∠AEC	13. Theorem 14
14. However, ∠ADC cannot be congruent to ∠AEC since m∠ADC > m∠AEC.	14. Theorem 37
15. Point D cannot be inside ⊙Q since, if it was, ∠ADC would not be supplementary to ∠ABC.	15. Contradiction of given that ∠ABC and ∠ADC are supplementary angles.
16. Point D must be on ⊙Q.	16. A simple plain closed curve divides the points in the plane into 3 distinct sets, points inside, points outside, and points on the curve.

2. $x = 90^\circ$ (Corollary 67a)
$y = 90^\circ$ (Corollary 67a)

$$m\overset{\frown}{BAD} = 360^\circ - m\overset{\frown}{BCD}$$
$$= 360^\circ - 136^\circ$$
$$= 224^\circ$$

$$z = \frac{1}{2} \cdot \overset{\frown}{BAD}$$
$$= \frac{1}{2} \cdot 224^\circ$$
$$= 112^\circ$$

or $\angle BCD$ and $\angle BAD$ are supplementary

$$m\angle BAD = 180^\circ - m\angle BCD$$

$$m\angle BAD = \frac{1}{2} \cdot m\overset{\frown}{BCD}$$
$$= \frac{1}{2} \cdot 136$$
$$= 68^\circ$$

$$68^\circ = 180^\circ - m\angle BCD$$
$$-112 = -m\angle BCD$$
$$112 = m\angle BCD$$
$$112^\circ = z$$

3. $\angle D$ and $\angle B$ are supplementary (Corollary 67b)

$$m\angle D + m\angle B = 180$$
$$102 + x = 180$$
$$x = 78$$

$\angle A$ and $\angle C$ are supplementary (Corollary 67b)

$$m\angle A + m\angle C = 180$$
$$y + 100 = 180$$
$$y = 80$$

4. $\angle A$ and $\angle C$ are supplementary (Corollary 67b)

$$m\angle A + m\angle C = 180$$
$$115 + x = 180$$
$$x = 65$$

$\angle B$ and $\angle D$ are supplementary (Corollary 67b)

$$m\angle B + m\angle D = 180$$
$$y + y = 180$$
$$2y = 180$$
$$y = 90$$

5. $4x + 24y = 180$ (Corollary 67b)

$$\frac{1}{4}\left(4x + 24y\right) = 180 \cdot \frac{1}{4}$$

$x + 6y = 45$ (Equation 1)

(Equation 2) $14x + 9y = 180$
(1) by –14 $-14x - 84y = -630$

$$-75y = -450$$
$$y = 6$$

$$4x + 24y = 180$$
$$4x + 24(6) = 180$$
$$4x + 144 = 180$$
$$4x = 36$$
$$x = 9$$

6. $2x + 26y = 180$ (Corollary 67b)

$$\frac{1}{2}(2x + 26y = 180)$$
$$x + 13y = 90$$

$$3x + 21y = 180$$
$$\frac{1}{3}(3x + 21y = 180)$$
$$x + 7y = 60$$

$$x + 13y = 90$$
$$-x - 7y = -60$$

$$6y = 30$$
$$y = 5$$

$$2x + 26y = 180$$
$$2x + 26\left(5\right) = 180$$
$$2x + 130 = 180$$
$$2x = 50$$
$$x = 25$$

7. $m\angle BAC = \frac{1}{2}m\overset{\frown}{BC}$, $45 = \frac{1}{2}m\overset{\frown}{BC}$, $90 = m\overset{\frown}{BC}$

$m\angle ACD = \frac{1}{2}m\overset{\frown}{AD}$, $40 = \frac{1}{2}m\overset{\frown}{AD}$, $80 = m\overset{\frown}{AD}$

$m\angle CDB = \frac{1}{2}m\overset{\frown}{BC} = \frac{1}{2}\cdot 90 = 45$

$m\angle ABD = \frac{1}{2}m\overset{\frown}{AD} = \frac{1}{2}\cdot 80 = 40$

$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$, $48 = \frac{1}{2}m\overset{\frown}{AB}$, $96 = m\overset{\frown}{AB}$

$m\angle ACB = \frac{1}{2}m\overset{\frown}{AB} = \frac{1}{2}\cdot 96 = 48$

$$m\overset{\frown}{BC} + m\overset{\frown}{AD} + m\overset{\frown}{AB} + m\overset{\frown}{CD} = 360$$
$$90 + 80 + 96 + m\overset{\frown}{CD} = 360$$
$$266 + m\overset{\frown}{CD} = 360$$
$$m\overset{\frown}{CD} = 94$$

$m\angle CAD = \frac{1}{2}m\overset{\frown}{CD} = \frac{1}{2}\cdot 94 = 47$

$m\angle CBD = \frac{1}{2}m\overset{\frown}{CD} = \frac{1}{2}\cdot 94 = 47$

$m\angle BAD = m\angle BAC + m\angle CAD = 45 + 47 = 92$

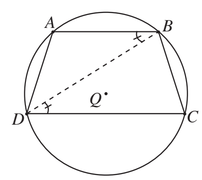
$m\angle ADC = m\angle ADB + m\angle CDB = 48 + 45 = 93$

$m\angle BCD = m\angle ACD + m\angle ACB = 40 + 48 = 88$

$m\angle ABC = m\angle ABD + m\angle CBD = 40 + 47 = 87$

8. Prove: If a trapezoid is inscribed in a circle, then it is an isosceles trapezoid.

Given: Trapezoid ABCD is inscribed in ⊙Q
Prove: Trapezoid ABCD is isosceles



STATEMENT	REASONS
1. Trapezoid ABCD is inscribed in ⊙Q.	1. Given
2. $\overline{AB} \parallel \overline{DC}$	2. Definition of a Trapezoid
3. Draw $\overline{BD}$.	3. Postulate 2 - For any two points there is exactly one line (segment) containing them.
4. $\angle ABD \cong \angle CDB$	4. Theorem 16 - if two parallel lines are cut by a transversal, then alternate interior angles are conruent.
5. $m\angle ABD = \frac{1}{2}m\overset{\frown}{AD}$	5. Theorem 67 - If you have an inscribed angle of a circle, then the measure of that angle is one-half the measure of the intercepted arc.
6. $m\angle CDB = \frac{1}{2}m\overset{\frown}{BC}$	6. Theorem 67
7. $m\angle ABD = m\angle CDB$	7. Definition of Congruent Angle
8. $\frac{1}{2}m\overset{\frown}{AD} = \frac{1}{2}m\overset{\frown}{BC}$	8. Substitution
9. $m\overset{\frown}{AD} = m\overset{\frown}{BC}$	9. Multiplication Property of Equality
10. $\overset{\frown}{AD} \cong \overset{\frown}{BC}$	10. Definition of Congruent Arcs
11. $\overline{AD} \cong \overline{BC}$	11. Theorem 82 - If two minor arcs are congruent then their chords are congruent.
12. Trapezoid ABCD is an isosceles trapezoid.	12. Definition of Isosceles Trapezoid - A trapezoid with congruent non-parallel sides. Q.E.D.

9. $m\angle ABC = 90$ (Corollary 67a)
$m\angle ADC = 90$ (Corollary 67a)

$$m\angle BAD + m\angle BCD = 180 \text{ (Corollary 67b)}$$
$$3x + x = 180$$
$$4x = 180$$
$$x = 45$$

$$m\angle BAD = 3x = 3(45) = 135$$

$$m\angle BCD = x = 45$$

10. The inscribed square is a rhombus, so diagonal $\overline{AC}$ bisects $\angle BAC$. $m\angle BAC = 90$, so $m\angle BAC = 45$.
$\triangle AEQ$ is therefore a 45-45-90 triangle. Let AE and QE equal x. AQ is then $x\sqrt{2}$.

$$\frac{QE}{QA} = \frac{x}{x\sqrt{2}} = \frac{x \cdot \sqrt{2}}{x\sqrt{2} \cdot \sqrt{2}} = \frac{x\sqrt{2}}{x \cdot 2} = \frac{\sqrt{2}}{2}$$

11. **a)** 120
b) 90
c) 60
d) 45

12.

STATEMENT	REASONS
1. Quadrilateral XYWZ is cyclic.	1. Given
2. $\overline{ZY}$ is a diameter of $\odot Q$.	2. Given
3. $\overline{XY} \parallel \overline{ZW}$	3. Given
4. $\angle XYZ \cong \angle WZY$	4. Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are conruent.
5. $\angle ZXY$ is a right angle.	5. Corollary 67a - If you have an angle inscribed in a semicircle, then that angle must be a right angle.
6. $\triangle ZXY$ is a right triangle.	6. Definition of Right Triangle
7. $\angle YWZ$ is a right angle.	7. Corollary 67a
8. $\triangle YWZ$ is a right triangle.	8. Definition of Right Triangle
9. $\triangle ZXY \cong \triangle YWZ$	9. Postulate Corollary 13e - If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the two right triangles are congruent. (HA)
10. $\overline{XZ} \cong \overline{WY}$	10. C.P.C.T.C.
11. $\overparen{XZ} \cong \overparen{WY}$	11. Theorem 81 - If two chords of a circle are congruent, then their minor arcs are congruent.
12. $\overparen{ZXY}$ is a semicircle.	12. Definition of Semicircle
13. $m\overparen{ZXY} = 180$	13. Definition of Semicircle
14. $\overparen{YWZ}$ is a semicircle.	14. Definition of Semicircle
15. $m\overparen{YWZ} = 180$	15. Definition of Semicircle
16. $m\overparen{ZXY} = m\overparen{YWZ}$	16. Substitution
17. $m\overparen{ZX} + m\overparen{XY} = m\overparen{ZXY}$	17. Postulate 8 - Circle - Arc Addition Assumption
18. $m\overparen{YW} + m\overparen{WZ} = m\overparen{YWZ}$	18. Postulate 8
19. $m\overparen{ZX} + m\overparen{XY} = m\overparen{YW} + m\overparen{WZ}$	19. Substitution
20. $m\overparen{XY} = m\overparen{WZ}$	20. Subtraction Property of Equality
21. $\overparen{XY} \cong \overparen{WZ}$	21. Definition of Congruent Arcs

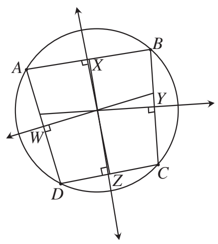
13. **a)** $m\angle AQB = 90$

b)
$$(AQ)^2 + (BQ)^2 = (AB)^2$$
$$(10)^2 + (10)^2 = (AB)^2$$
$$100 + 100 = (AB)^2$$
$$200 = (AB)^2$$
$$\pm\sqrt{200} = AB \quad \text{(AB cannot be negative)}$$
$$\sqrt{200} = AB$$
$$\sqrt{100 \cdot 2} = AB$$
$$10\sqrt{2} = AB$$

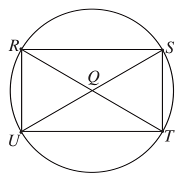
c) Distance from point Q to AB $= \frac{1}{2} \cdot AB$
$$= \frac{1}{2} \cdot 10\sqrt{2}$$
$$= 5\sqrt{2}$$

14. Always - Theorem 84
15. Always - Theorem 84
16. Not Always - Only in some special parallelograms are opposite angles supplementary (Example: Square and rectangle).
17. Not Always - If one pair of opposite angles are right angles, the kite is cyclic.
18. Not Always - The rhombus must be a square.
19. Always - Theorem 84

20. Points X, Y, Z and W are midpoints of the sides of quadrilateral ABCD.



21. Rectangle RSTU is inscribed in circle Q.
In a rectangle, $\overline{RS} \cong \overline{TU}$, $\overline{ST} \cong \overline{UR}$, and $\overline{RT} \cong \overline{SU}$.



Ptolomy's Theorem: (RS)(TU) + (ST)(UR) = (RT)(SU)
Use Substitution: (RS)(RS) + (ST)(ST) = (RT)(RT)
For $\triangle RST$ $(RS)^2 + (ST)^2 = (RT)^2$