

Unit VI - Circles

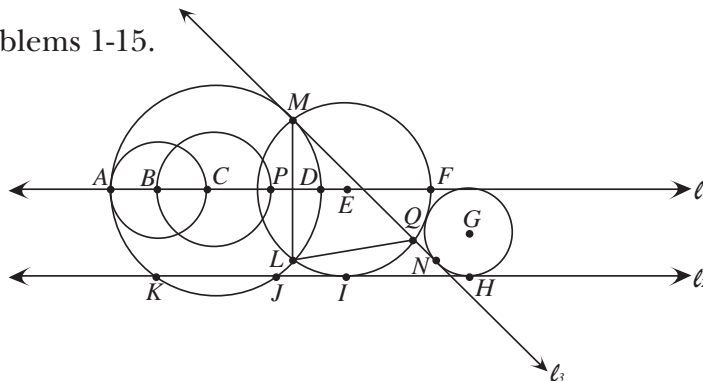
Part A - Fundamental Terms

Lesson 1 - Lines and Segments

Lesson 2 - Arcs and Angles

Lesson 3 - Circle Relationships

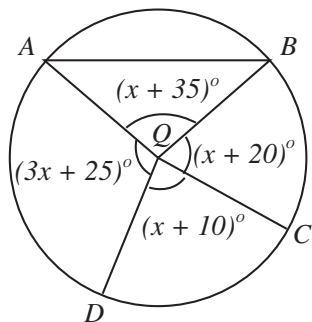
Use the given figure for problems 1-15.



1. l_3 is a common internal tangent of $\odot G$ and $\odot C$. (with diameter \overline{AD})
2. \widehat{KJ} and \widehat{LM} are minor arcs of $\odot C$.
3. \overline{AD} is a diameter of $\odot C$.
4. $\odot E$ and $\odot G$ are externally tangent circles.
5. $\odot C$ (with diameter \overline{BP}) and $\odot C$ (with diameter \overline{AD}) are concentric circles.
6. \overline{ML} is a common chord of $\odot E$ and $\odot C$ (with diameter \overline{AD}).
7. l_2 is a common external tangent to \odot G and \odot E.
8. $\triangle MLQ$ is inscribed in $\odot E$.
9. l_2 is a secant of $\odot C$. (with diameter \overline{AD})
10. \widehat{ML} is a minor arc of $\odot E$.
11. $\odot E$ is circumscribed about $\triangle MLQ$.
12. \overline{EF} is a radius of $\odot E$.
13. $\angle MQL$ is an inscribed angle of $\odot E$.
14. \widehat{MLQ} is a major arc of $\odot E$.
15. \widehat{AKD} is a semicircle of $\odot C$ (with diameter \overline{AD}).

—Continued—

Use the given figure for problems 16 - 20. (Note: the open phrases are the measures of the central angles indicated)



16. $x = \underline{\hspace{2cm} 45 \hspace{2cm}}$

$$\begin{aligned} (x+35)+(x+20)+(x+10)+(3x+25) &= 360 \\ 6x+90 &= 360 \\ 6x &= 360 \\ x &= 45 \end{aligned}$$

17. $m\widehat{AD} = \underline{\hspace{2cm} 160 \hspace{2cm}}$

$$\begin{aligned} m\widehat{AD} &= 3x+25 \\ &= 3(45)+25 \\ &= 135+25 \\ &= 160 \end{aligned}$$

18. $m\widehat{BC} = \underline{\hspace{2cm} 65 \hspace{2cm}}$

$$\begin{aligned} m\widehat{BC} &= x+20 \\ &= 45+20 \\ &= 65 \end{aligned}$$

19. $m\widehat{ABC} = \underline{\hspace{2cm} 145 \hspace{2cm}}$

$$\begin{aligned} m\widehat{ABC} &= (x+35)+(x+20) \\ &= 45+35+45+20 \\ &= 145 \end{aligned}$$

20. $m\widehat{BCA} = \underline{\hspace{2cm} 280 \hspace{2cm}}$

$$\begin{aligned} m\widehat{BCA} &= (x+20)+(x+10)+(3x+25) \\ &= 5(45)+55 \\ &= 225+55 \\ &= 280 \end{aligned}$$

21. $m\angle ABQ = \underline{\hspace{2cm} 50 \hspace{2cm}}$

$m\angle BQA = 80^\circ$ and
 $\triangle BQA$ is Isosceles.
 Base angles of an isosceles
 triangle are congruent.

Unit VI - Circles

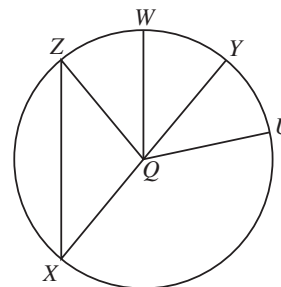
Part A - Fundamental Terms

Lesson 1 - Lines and Segments

Lesson 2 - Arcs and Angles

Lesson 3 - Circle Relationships

Use the given diagram to complete problems 1-18.
 (Note: $m\angle WQY = 30^\circ$, $m\angle UQY = 30^\circ$, $m\angle XQZ = 100^\circ$)

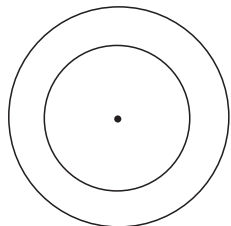


- Name five central angles which appear to be acute angles. $\angle UQY, \angle YQW, \angle WQZ, \angle UQW, \angle YQZ$
- Name four central angles which appear to be obtuse angles. $\angle XQZ, \angle XQW, \angle ZQU, \angle XQU$
- Name four minor arcs which are not a combination of two arcs. $\widehat{UY}, \widehat{YW}, \widehat{WZ}, \widehat{XZ}$ (also \widehat{UX})
- Name the largest major arc which is not a combination of more than two arcs. \widehat{ZXU}
- Name a semicircle of $\odot Q$. \widehat{XZY} (or \widehat{XUY})
- Name a pair of congruent arcs. \widehat{WY} and \widehat{UY}
- Name a pair of congruent chords. \overline{WY} and \overline{UY}
- What is $m\widehat{UY}$? 30
- What is $m\widehat{ZW}$? 50
- What is $m\widehat{ZXU}$? 250 (360 - 30 - 30 - 50)
- $m\widehat{WY} + m$ \widehat{WZ} = $m\widehat{YZ}$
- $m\widehat{XZ} + m$ \widehat{ZU} = $m\widehat{XWU}$
- \overline{XY} is a diameter of $\odot Q$.
- \overline{QW} is a radius of $\odot Q$.
- $\overline{QZ} \cong \overline{QY}$.
- Point X is connected to point Z, so \overline{XZ} is a chord of $\odot Q$.
- $\angle ZXY$ is an inscribed angle.
- $\angle XQZ$ is a central angle.

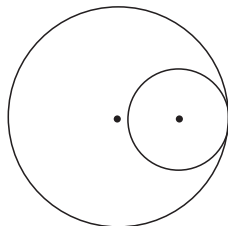
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19. Sketch two circles that are:

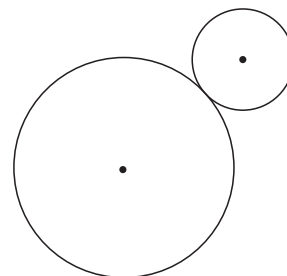
a) concentric



b) internally tangent

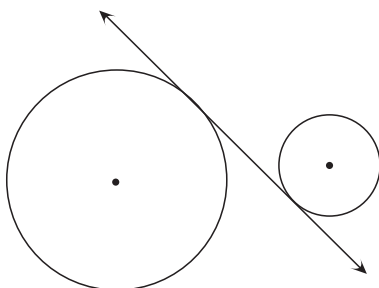


c) externally tangent

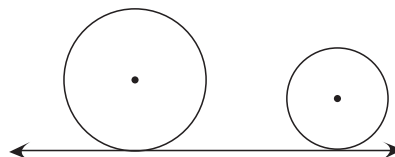


20. Sketch two circles with a:

a) common internal tangent

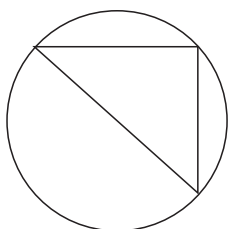


b) common external tangent

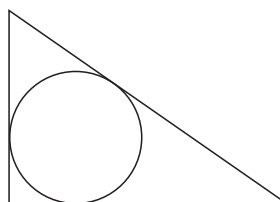


21. Sketch a circle:

a) circumscribed about a triangle



b) inscribed in a triangle



Unit VI - Circles

Part B - Angle and Arc Relationships

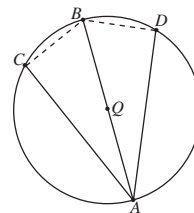
Lesson 1 - Theorem 65 - "If, in the same circle, or in congruent circles, two central angles are congruent, then their intercepted minor arcs are congruent."

Theorem 66 - "If, in the same circle, or in congruent circles, two minor arcs are congruent, then the central angles which intercept those minor arcs are congruent."

Lesson 2 - Theorem 67 - "If you have an inscribed angle of a circle, then the measure of that angle is one-half the measure of its intercepted arc."

Lesson 3 - Theorem 68 - "If, in a circle, you have an angle formed by a secant ray, and a tangent ray, both drawn from a point on the circle, then the measure of that angle, is one-half the measure of the intercepted arc."

1. Given: \overline{AB} is a diameter of $\odot Q$.
 $\overline{AC} \cong \overline{AD}$

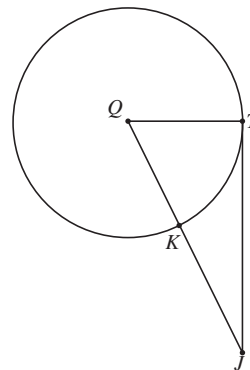


Prove: $\widehat{CB} \cong \widehat{DB}$

STATEMENT	REASON
1. Draw \overline{CB}	1. Postulate 2 - For any two different points, there is exactly one line containing them.
2. Draw \overline{DB}	2. Postulate 2
3. \overline{AB} is a diameter of circle Q	3. Given
4. \widehat{ADB} is a semicircle	4. Definition of Semicircle - An arc of a circle is a semicircle, if and only if, it is an arc whose endpoints are the endpoints of a diameter of the circle.
5. \widehat{ACB} is a semicircle	5. Definition of Semicircle
6. $\angle ADB$ is a right angle	6. Corollary 67a - If you have an angle inscribed in a semicircle then it is a right angle.
7. $\angle ACB$ is a right angle	7. Corollary 67a
8. $\triangle ADB$ is a right triangle	8. Definition of Right Triangle
9. $\triangle ACB$ is a right triangle	9. Definition of Right Triangle
10. $\overline{AC} \cong \overline{AD}$	10. Given
11. $\overline{AB} \cong \overline{AB}$	11. Reflexive Property of Congruence
12. $\triangle ADB \cong \triangle ACB$	12. Postulate Corollary 13c - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and one leg of another right triangle then the two right triangles are congruent.
13. $\angle CAB \cong \angle DAB$	13. C.P.C.T.C.
14. $m\angle CAB = m\angle DAB$	14. Definition of Congruent Angles
15. $m\angle CAB = \frac{1}{2}m\widehat{CB}$	15. Theorem 67 - If you have an inscribed angle of a circle, then the measure of the angle is one-half the measure of the intercepted arc.
16. $m\angle DAB = \frac{1}{2}m\widehat{DB}$	16. Theorem 67
17. $\frac{1}{2}m\widehat{CB} = \frac{1}{2}m\widehat{DB}$	17. Substitution
18. $m\widehat{CB} = m\widehat{DB}$	18. Multiplication Property of Equality
19. $\widehat{CB} \cong \widehat{DB}$	19. Definition of Congruent Arcs

—Continued—

2. Use the figure to the right to complete the following statements. In the figure, \overline{JT} is tangent to $\odot Q$ at point T.



- a) If $QT = 6$ and $JQ = 10$, then $JT =$ _____ 8

Since $\overline{QT} \perp \overline{JT}$ by Corollary 68a, $\triangle QTJ$ is a right triangle. $a^2 + b^2 = c^2$

$$(QT)^2 + (JT)^2 = (JQ)^2$$

$$6^2 + (JT)^2 = 10^2$$

$$36 + (JT)^2 = 100$$

$$(JT)^2 = 64$$

$$JT = 8$$

- b) If $QT = 8$ and $JT = 15$, then $JQ =$ _____ 17

Since $\overline{QT} \perp \overline{JT}$ by Corollary 68a, $\triangle QTJ$ is a right triangle. $a^2 + b^2 = c^2$

$$(QT)^2 + (JT)^2 = (JQ)^2$$

$$8^2 + 15^2 = (JQ)^2$$

$$64 + 225 = (JQ)^2$$

$$289 = (JQ)^2$$

$$17 = JQ$$

- c) If $m\angle JQT = 60$ and $QT = 6$, then $JQ =$ _____ 12

Since $\overline{QT} \perp \overline{JT}$, $\triangle QTJ$ is a right triangle. If $m\angle JQT = 60$, $\triangle QTJ$ is a 30-60-90 triangle, and \overline{QT} is the short leg of the triangle opposite the 30 degree angle. $QT = \frac{1}{2}JQ$ in a 30-60-90 triangle.

$$QT = \frac{1}{2}JQ$$

$$6 = \frac{1}{2}JQ$$

$$12 = JQ$$

- d) If $JQ = 9$ and $KQ = 8$, then $JT =$ _____ 15

Since $\overline{QT} \perp \overline{JT}$, $\triangle QTJ$ is a right triangle. $a^2 + b^2 = c^2$

$$(QT)^2 + (JT)^2 = (JQ)^2$$

$$8^2 + (JT)^2 = (JK + KQ)^2$$

$$64 + (JT)^2 = (9 + 8)^2$$

$$64 + (JT)^2 = (17)^2$$

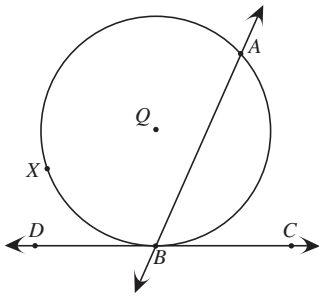
$$64 + (JT)^2 = 289$$

$$(JT)^2 = 225$$

$$JT = 15$$

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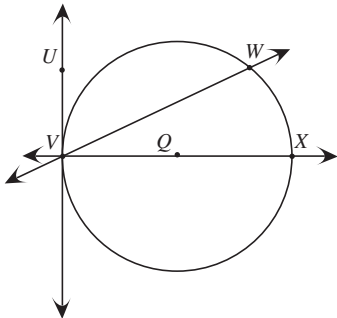
3.



In the figure to the left, if $m\widehat{AXB} = 220$,
then $m\angle ABC = \underline{70}$ and $m\angle DBA = \underline{110}$.

$$\begin{aligned} m\widehat{AB} &= 360 - m\widehat{AXB} & m\angle ABC &= \frac{1}{2} \cdot m\widehat{AB} & m\angle DBA &= \frac{1}{2} \cdot m\widehat{AXB} \\ &= 360 - 220 & &= \frac{1}{2} \cdot 140 & &= \frac{1}{2} \cdot 220 \\ &= 140 & &= \cancel{2} \cdot 70 & &= \cancel{2} \cdot 110 \\ & & &= 70 & &= 110 \end{aligned}$$

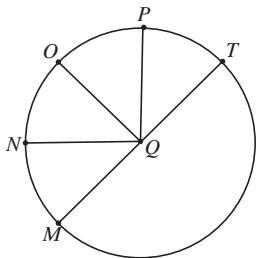
4.



In the figure to the left, if $m\widehat{VW} = 114$,
then $m\angle WVX = \underline{33}$ and $m\angle UVW = \underline{57}$.

$$\begin{aligned} m\widehat{WX} &= 180 - m\widehat{VW} & m\angle WVX &= \frac{1}{2} \cdot m\widehat{WX} & m\angle UVW &= \frac{1}{2} \cdot m\widehat{VW} \\ &= 180 - 114 & &= \frac{1}{2} \cdot 66 & &= \frac{1}{2} \cdot 114 \\ &= 66 & &= \cancel{2} \cdot 33 & &= \cancel{2} \cdot 57 \\ & & &= 33 & &= 57 \end{aligned}$$

5.

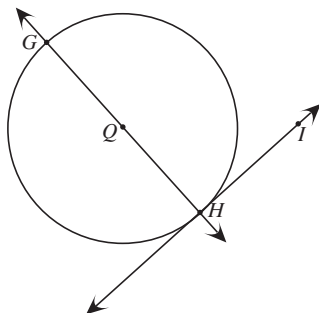


In the figure to the left, if $m\angle MQN = m\angle TQP = 30$, and
 $m\widehat{NO} = 60$, then $m\angle NQO = \underline{60}$ and $m\widehat{OP} = \underline{60}$.

$$\begin{aligned} m\angle OQN &= m\widehat{NO} & m\angle PQO &= 180 - (m\angle TQP + m\angle OQN + m\angle NQM) \\ &= 60 & &= 180 - (30 + 60 + 30) \\ & & &= 180 - 120 \\ & & &= 60 \end{aligned}$$

$$\begin{aligned} m\widehat{OP} &= m\angle PQO \\ &= 60 \end{aligned}$$

6.

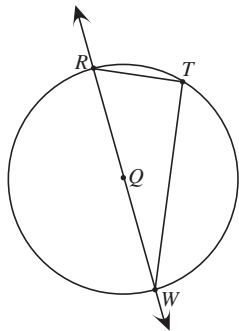


In the figure to the left, if \overleftrightarrow{HI} is tangent to $\odot Q$ at the
endpoint H of diameter \overline{GH} , then $m\angle GHI = \underline{90}$

$$m\angle GHI = 90 \quad \text{Corollary 68a}$$

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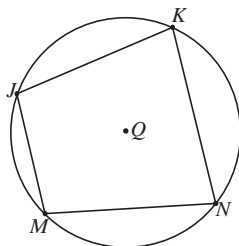
7.



In the figure to the left, if $m\widehat{RT} = 56$, then $m\angle RTW = \underline{\quad 90 \quad}$,
 $m\angle RWT = \underline{\quad 28 \quad}$ and $m\angle TRW = \underline{\quad 62 \quad}$.

$$\begin{aligned}
 m\angle RTW &= 90 && \text{Corollary 68a} && m\angle RWT &= \frac{1}{2} \cdot m\widehat{RT} \\
 m\angle TRW &= 180 - (m\angle RTW + m\angle RWT) && && &= \frac{1}{2} \cdot 56 \\
 &= 180 - (90 + 28) && && &= \cancel{2} \cdot 28 \\
 &= 180 - 118 && && &= \cancel{2} \\
 &= 62 && && &= 28
 \end{aligned}$$

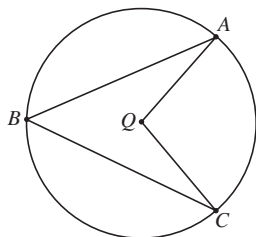
8.



In the figure to the left, if $m\angle J = 105$, and $m\angle K = 80$
 then $m\angle M = \underline{\quad 100 \quad}$ and $m\angle N = \underline{\quad 75 \quad}$.

$$\begin{aligned}
 m\angle M &= 180 - m\angle K && m\angle N &= 180 - m\angle J \\
 &= 180 - 80 && &= 180 - 105 \\
 &= 100 && &= 75
 \end{aligned}$$

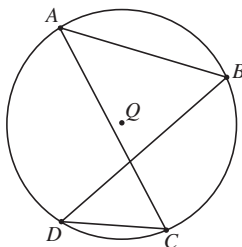
9.



In the figure to the left, if $m\angle AQC = 94$,
 then $m\angle ABC = \underline{\quad 47 \quad}$ and $m\widehat{AC} = \underline{\quad 94 \quad}$.

$$\begin{aligned}
 m\angle ABC &= \frac{1}{2} \cdot m\widehat{AC} && m\widehat{AC} &= m\angle AQC \\
 &= \frac{1}{2} \cdot 94 && &= 94 \\
 &= \cancel{2} \cdot 47 && & \\
 &= 47 && &
 \end{aligned}$$

10.



In the figure to the left, if $m\widehat{AD} = 116$,
 then $m\angle ABD = \underline{\quad 58 \quad}$ and $m\angle DCA = \underline{\quad 58 \quad}$.

$$\begin{aligned}
 m\angle ABD &= m\angle DCA \\
 &= \frac{1}{2} \cdot m\widehat{AD} \\
 &= \frac{1}{2} \cdot 116 \\
 &= \cancel{2} \cdot 58 \\
 &= 58
 \end{aligned}$$

Unit VI - Circles

Part B - Angle and Arc Relationships

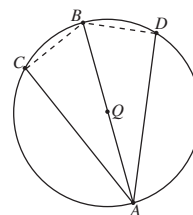
Lesson 1 - Theorem 65 - "If, in the same circle, or in congruent circles, two central angles are congruent, then their intercepted minor arcs are congruent."

Theorem 66 - "If, in the same circle, or in congruent circles, two minor arcs are congruent, then the central angles which intercept those minor arcs are congruent."

Lesson 2 - Theorem 67 - "If you have an inscribed angle of a circle, then the measure of that angle is one-half the measure of its intercepted arc."

Lesson 3 - Theorem 68 - "If, in a circle, you have an angle formed by a secant ray, and a tangent ray, both drawn from a point on the circle, then the measure of that angle, is one-half the measure of the intercepted arc."

1. Given: \overline{AB} is a diameter of $\odot Q$.
 \overline{AB} bisects \widehat{CD}

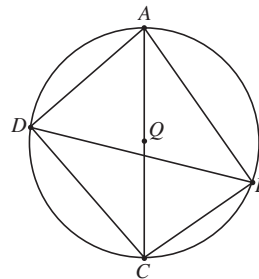


Prove: $\overline{AC} \cong \overline{AD}$

STATEMENT	REASON
1. Draw \overline{CB}	1. Postulate 2 - For any two different points, there is exactly one line containing them.
2. Draw \overline{DB}	2. Postulate 2
3. \overline{AB} is a diameter of circle Q	3. Given
4. \widehat{ADB} is a semicircle	4. Definition of Semicircle - An arc of a circle is a semicircle, if and only if, it is an arc whose endpoints are the endpoints of a diameter of the circle.
5. \widehat{ACB} is a semicircle	5. Definition of Semicircle
6. $\angle ADB$ is a right angle	6. Corollary 67a - If you have an angle inscribed in a semicircle then it is a right angle.
7. $\angle ACB$ is a right angle	7. Corollary 67a
8. $\triangle ADB$ is a right triangle	8. Definition of Right Triangle
9. $\triangle ACB$ is a right triangle	9. Definition of Right Triangle
10. \overline{AB} bisects \widehat{CD}	10. Given
11. $\widehat{CB} \cong \widehat{DB}$	11. Definition of Arc Bisector
12. $m\widehat{CB} = m\widehat{DB}$	12. Definition of Congruent Arcs
13. $\frac{1}{2}m\widehat{CB} = \frac{1}{2}m\widehat{DB}$	13. Multiplication Property of Equality
14. $m\angle BAC = \frac{1}{2}m\widehat{CB}$	14. Theorem 67 - If you have an inscribed angle of a circle, then the measure of the angle is one-half the measure of the intercepted arc.
15. $m\angle BAD = \frac{1}{2}m\widehat{DB}$	15. Theorem 67
16. $m\angle BAC = m\angle BAD$	16. Substitution
17. $\angle BAC \cong \angle BAD$	17. Definition of Angle Congruence
18. $\overline{AB} \cong \overline{AB}$	18. Reflexive Property of Congruence
19. $\triangle ACB \cong \triangle ADB$	19. Postulate Corollary 13e - If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the two right triangles are congruent.
20. $\overline{AC} \cong \overline{AD}$	20. C.P.C.T.C.

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2. In the figure to the right, quadrilateral ABCD is inscribed in $\odot Q$. Additionally, \overline{AC} is a diameter of $\odot Q$, the $m\angle ABD = 44$, and $m\widehat{AB} = 116$. Use this information for the following:



- a) Find $m\angle ABC$ 90

$m\angle ABC = 90$ since it is an angle inscribed in a semicircle.

- b) Find $m\angle BCA$ 58

$$\begin{aligned} m\angle BCA &= \frac{1}{2} \cdot m\widehat{AB} \\ &= \frac{1}{2} \cdot 116 \\ &= \frac{\cancel{2} \cdot 58}{\cancel{2}} \\ &= 58 \end{aligned}$$

- c) Find $m\angle CDB$ 32

$$\begin{aligned} m\angle BAC &= 180 - (m\angle ABC + m\angle BCA) & m\angle BAC &= \frac{1}{2} \cdot m\widehat{BC} \\ &= 180 - (90 + 58) & m\angle CDB &= \frac{1}{2} \cdot m\widehat{BC} \\ &= 180 - 148 & m\angle BAC &= m\angle CDB \\ &= 32 & 32 &= m\angle CDB \end{aligned}$$

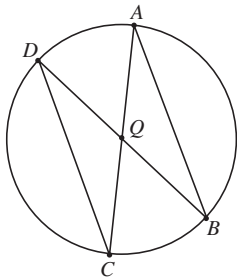
Corollary 67c - If two inscribed angles intercept the same or congruent arcs, then the angles are congruent.

- d) Find $m\angle BAD$ 78

$$\begin{aligned} \text{Since } m\angle CDB = 32 & & m\angle BAD &= \frac{1}{2} (m\widehat{BC} + m\widehat{DC}) \\ m\widehat{BC} = 64 & & &= \frac{1}{2} (64 + 92) \\ \text{Since } m\angle DBC = 46 & & &= \frac{1}{2} (156) \\ m\widehat{DC} = 92 & & &= 78 \end{aligned}$$

—Continued—

3.



In the figure to the left, if $m\widehat{AD} = 68$,
then $m\angle ABD = \underline{\quad 34 \quad}$ and $m\angle ACD = \underline{\quad 34 \quad}$.

$$m\angle ABD = m\angle ACD$$

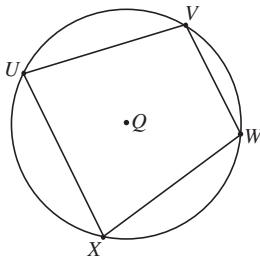
$$m\angle ACD = \frac{1}{2} \cdot m\widehat{AD}$$

$$= \frac{1}{2} \cdot 68$$

$$= \frac{\cancel{2} \cdot 34}{\cancel{2}}$$

$$= 34$$

4.



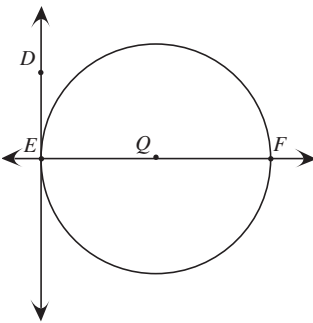
In the figure to the left, if $m\angle V = 94$,
then $m\angle UXW = \underline{\quad 86 \quad}$.

$$m\angle UXW = 180 - m\angle UVW$$

$$= 180 - 94$$

$$= 86$$

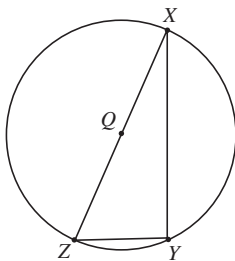
5.



In the figure to the left, if \overleftrightarrow{DE} is tangent to $\odot Q$ at point E
of diameter \overline{EF} , then $m\angle DEF = \underline{\quad 90 \quad}$.

Corollary 68a

6.

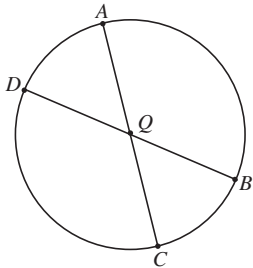


In the figure to the left, if \overline{XZ} is a diameter,
then $m\angle XYZ = \underline{\quad 90 \quad}$.

Corollary 67a

—Continued—

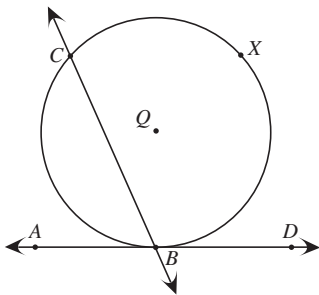
7.



In the figure to the left, if $m\widehat{BC} = 37$,
then $m\widehat{AD} = \underline{\quad 37 \quad}$.

Congruent central angles give congruent intercepted arcs.

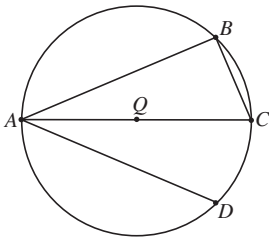
8.



In the figure to the left, if $m\angle ABC = 64$,
then $m\widehat{BXC} = \underline{\quad 232 \quad}$.

$$\begin{aligned} m\angle ABC &= \frac{1}{2} \cdot m\widehat{BC} & m\widehat{BXC} &= 360 - m\widehat{BC} \\ 64 &= \frac{1}{2} \cdot m\widehat{BC} & &= 360 - 128 \\ 128 &= m\widehat{BC} & &= 232 \end{aligned}$$

9.

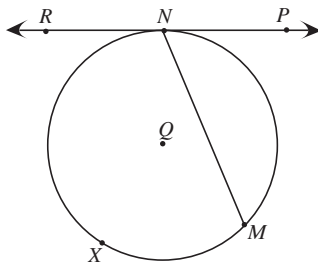


In the figure to the left, if $m\angle BAC = 22$, and $m\widehat{CD} = 44$,
then $m\angle DAC = \underline{\quad 22 \quad}$, $m\angle ACB = \underline{\quad 68 \quad}$
and $m\widehat{BC} = \underline{\quad 44 \quad}$.

$$\begin{aligned} m\widehat{AB} &= 180 - m\widehat{BC} \\ &= 180 - 44 \\ &= 136 \end{aligned}$$

$$\begin{aligned} m\angle DAC &= \frac{1}{2} \cdot m\widehat{DC} & m\angle BAC &= \frac{1}{2} \cdot m\widehat{BC} & m\angle ACB &= \frac{1}{2} \cdot m\widehat{AB} \\ &= \frac{1}{2} \cdot 44 & 22 &= \frac{1}{2} \cdot m\widehat{BC} & &= \frac{1}{2} \cdot 136 \\ &= \frac{\cancel{2} \cdot 22}{\cancel{2}} & 44 &= m\widehat{BC} & &= \frac{\cancel{2} \cdot 68}{\cancel{2}} \\ &= 22 & & & &= 68 \end{aligned}$$

10.



In the figure to the left, if $m\widehat{MXN} = 200$,
then $m\angle MNP = \underline{\quad 80 \quad}$

$$\begin{aligned} m\angle RNM &= \frac{1}{2} \cdot m\widehat{NXM} & m\angle MNP &= 180 - m\angle RNM \\ &= \frac{1}{2} \cdot 200 & &= 180 - 100 \\ &= \frac{\cancel{2} \cdot 100}{\cancel{2}} & &= 80 \\ &= 100 & & \end{aligned}$$

Unit VI - Circles

Part B - Angle and Arc Relationships

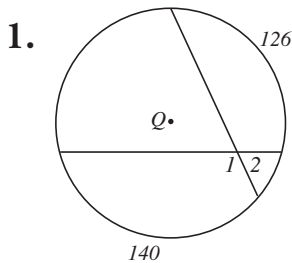
Lesson 4 - Theorem 69 - "If, for a circle, two secant lines intersect inside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle) is equal to one-half the sum of the measures of the arcs intercepted by the angle, and its vertical angle."

Theorem 70 - "If, for a circle, two secant lines intersect outside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle), is equal to one-half the difference of the measures of the arcs intercepted by the angle."

Lesson 5 - Theorem 71 - "If, for a circle, a secant line and a tangent line intersect outside a circle, then the measure of the angle formed, is equal to one-half the difference of the measures of the arcs intercepted by the angle."

Theorem 72 - "If, for a circle, two tangent lines intersect outside the circle, then the measure of the angle formed, is equal to one-half the difference of the measures of the arcs intercepted by the angle."

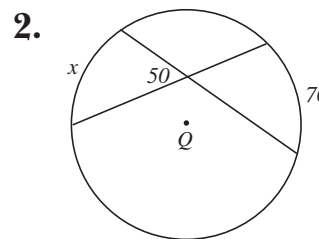
In problems 1-10, find the measure of each numbered angle or lettered arc in $\odot Q$.



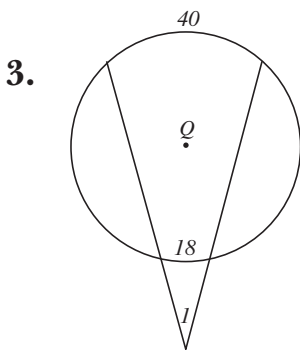
$$\begin{aligned} m\angle 2 &= 180 - m\angle 1 \\ &= 180 - 133 \\ &= 47 \end{aligned}$$

$$\begin{aligned} m\angle 1 &= \frac{133}{2} \\ m\angle 2 &= \frac{47}{2} \end{aligned}$$

$$\begin{aligned} m\angle 1 &= \frac{1}{2}(140 + 126) \\ &= \frac{1}{2} \cdot 266 \\ &= \frac{2 \cdot 133}{2} \\ &= 133 \end{aligned}$$

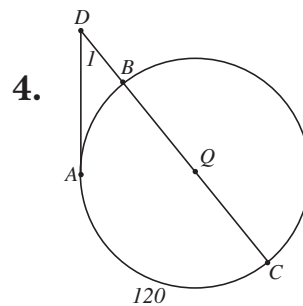


$$\begin{aligned} x &= \frac{30}{2} \\ 50 &= \frac{1}{2}(x + 70) \\ 100 &= x + 70 \\ 30 &= x \end{aligned}$$



$$m\angle 1 = \frac{11}{2}$$

$$\begin{aligned} m\angle 1 &= \frac{1}{2}(40 - 18) \\ &= \frac{1}{2} \cdot 22 \\ &= \frac{2 \cdot 11}{2} \\ &= 11 \end{aligned}$$

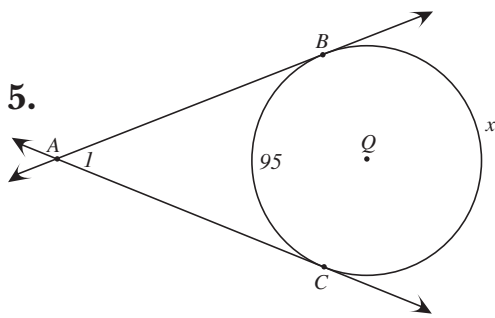


$$\begin{aligned} m\widehat{AB} &= 180 - 120 \\ &= 60 \end{aligned}$$

$$m\angle 1 = \frac{30}{2}$$

$$\begin{aligned} m\angle 1 &= \frac{1}{2}(m\widehat{AC} - m\widehat{AB}) \\ m\angle 1 &= \frac{1}{2}(120 - 60) \\ &= \frac{1}{2} \cdot 60 \\ &= \frac{2 \cdot 30}{2} \\ &= 30 \end{aligned}$$

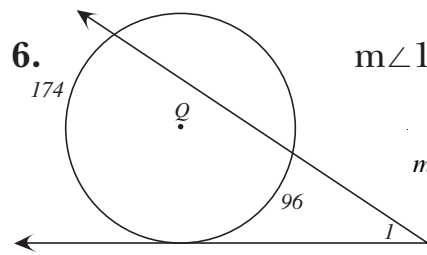
—Continued—



$m\angle 1 = \underline{85}$ $m\angle 1 = \frac{1}{2}(m\widehat{BXC} - m\widehat{BC})$

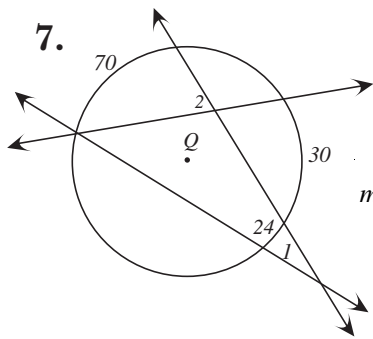
$m\widehat{BXC} = 360 - m\widehat{BC}$
 $= 360 - 95$
 $= 265$

$m\angle 1 = \frac{1}{2}(265 - 95)$
 $= \frac{1}{2} \cdot 170$
 $= \frac{\cancel{2} \cdot 85}{\cancel{2}}$
 $= 85$



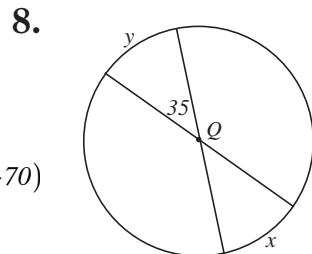
$m\angle 1 = \underline{39}$

$m\angle 1 = \frac{1}{2}(174 - 96)$
 $= \frac{1}{2} \cdot 78$
 $= \frac{\cancel{2} \cdot 39}{\cancel{2}}$
 $= 39$

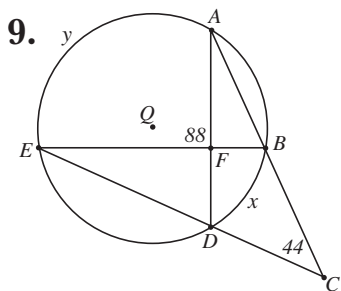


$m\angle 1 = \underline{23}$
 $m\angle 2 = \underline{50}$

$m\angle 1 = \frac{1}{2}(70 - 24)$ $m\angle 2 = \frac{1}{2}(30 + 70)$
 $= \frac{1}{2} \cdot 46$ $= \frac{1}{2} \cdot 100$
 $= \frac{\cancel{2} \cdot 23}{\cancel{2}}$ $= \frac{\cancel{2} \cdot 50}{\cancel{2}}$
 $= 23$ $= 50$



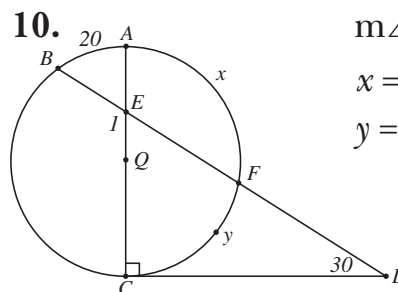
$y = \underline{35}$
 $x = \underline{35}$



$x = \underline{48}$
 $y = \underline{112}$
 $m\angle AFE = \frac{1}{2}(x + y)$
 $88 = \frac{1}{2}(x + y)$
 $176 = x + y$

$176 = y + x$ $48 - y = -x$
 $48 = y - x$ $-48 + y = x$
 $224 = 2y$ $-48 + 112 = x$
 $112 = y$ $64 = x$

$m\angle ACE = \frac{1}{2}(y - x)$
 $24 = \frac{1}{2}(y - x)$
 $48 = y - x$



$m\angle 1 = \underline{120}$
 $x = \underline{80}$
 $y = \underline{100}$

$\angle 1 = 180 - 60$
 $= 120$

$m\angle 1 = \frac{1}{2}(m\widehat{BC} + m\widehat{AF})$
 $120 = \frac{1}{2}(160 + x)$

$m\angle CED = 180 - (90 + 30)$
 $= 180 - 120$
 $= 60$

$y = 180 - x$ $120 = 80 + \frac{1}{2}x$
 $= 180 - 80$ $40 = \frac{1}{2}x$
 $= 100$ $80 = x$

Unit VI, Part B, Lessons 4&5, Quiz Form A
—Continued—

Name _____

In problems 11-14, determine which statements are true and which are false.

- 11.** No tangents of a circle are secants of that circle. *true*
- 12.** A point of tangency is outside the circle. *false*
- 13.** If a line contains a point inside a circle, it is a secant of that circle. *true*
- 14.** From a given point, there is only one tangent to a given circle. *false*

Unit VI - Circles

Part B - Angle and Arc Relationships

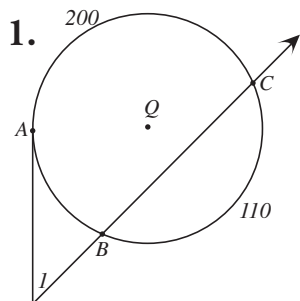
Lesson 4 - Theorem 69 - "If, for a circle, two secant lines intersect inside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle) is equal to one-half the sum of the measures of the arcs intercepted by the angle, and its vertical angle."

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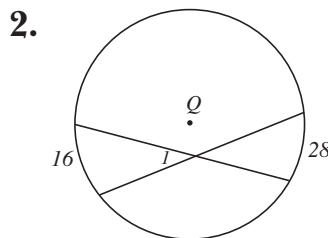
In problems 1-10, find the measure of each numbered angle or lettered arc in $\odot Q$.



$$m\angle I = \underline{\quad 75 \quad}$$

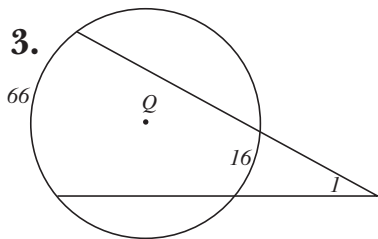
$$\begin{aligned} m\widehat{AB} &= 360 - (200 + 110) \\ &= 360 - 310 \\ &= 50 \end{aligned}$$

$$\begin{aligned} m\angle I &= \frac{1}{2}(200 + 50) \\ &= \frac{1}{2}(250) \\ &= \frac{\cancel{2} \cdot 125}{\cancel{2}} \\ &= 125 \end{aligned}$$



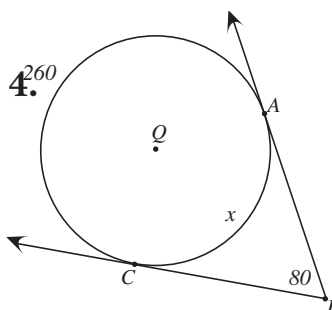
$$m\angle I = \underline{\quad 22 \quad}$$

$$\begin{aligned} m\angle I &= \frac{1}{2}(28 + 16) \\ &= \frac{1}{2}(44) \\ &= \frac{\cancel{2} \cdot 22}{\cancel{2}} \\ &= 22 \end{aligned}$$



$$m\angle I = \underline{\quad 25 \quad}$$

$$\begin{aligned} m\angle I &= \frac{1}{2}(66 - 16) \\ &= \frac{1}{2}(50) \\ &= \frac{\cancel{2} \cdot 25}{\cancel{2}} \\ &= 25 \end{aligned}$$



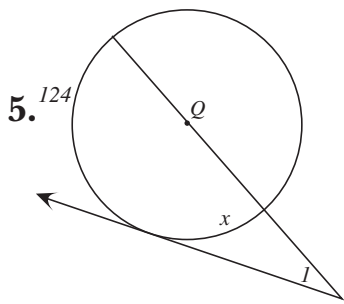
$$x = \underline{\quad 100 \quad}$$

$$\begin{aligned} 80 &= \frac{1}{2}(260 - x) \\ 80 &= 130 - \frac{1}{2}x \\ -50 &= -\frac{1}{2}x \end{aligned}$$

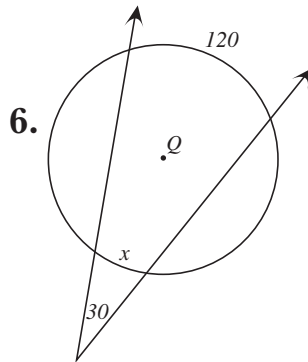
(another way)
 $x = 360 - 260$
 $x = 100$

$$100 = x$$

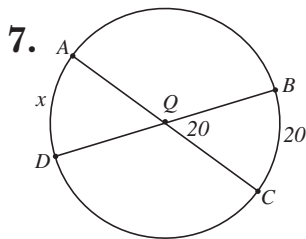
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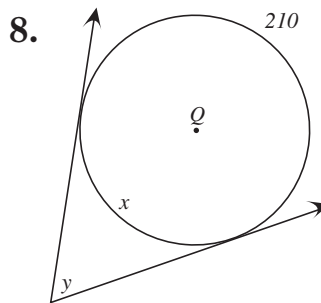
5. $m\angle l = \frac{1}{2}(124 - x)$
 $x = 56$
 $x = 180 - 124 = 56$
 $m\angle l = \frac{1}{2}(124 - 56) = \frac{1}{2}(68) = \frac{\cancel{2} \cdot 34}{\cancel{2}} = 34$



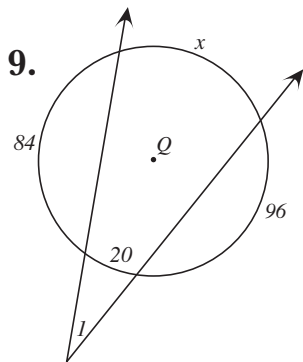
6. $x = 60$
 $30 = \frac{1}{2}(120 - x)$
 $30 = 60 - \frac{1}{2}x$
 $-30 = -\frac{1}{2}x$
 $60 = x$



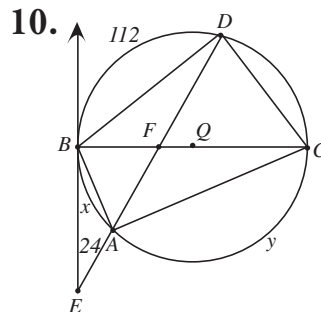
7. $x = 20$
 $x = 20$ (from a central angle)
 or
 $20 = \frac{1}{2}(x + 20)$
 $20 = \frac{1}{2}x + 10$
 $10 = \frac{1}{2}x$
 $20 = x$



8. $y = 30$
 $x = 150$
 $x = 360 - 210 = 150$
 $y = \frac{1}{2}(210 - 150) = \frac{1}{2}(60) = \frac{\cancel{2} \cdot 30}{\cancel{2}} = 30$



9. $m\angle l = \frac{1}{2}(84 + 96 - x)$
 $x = 160$
 $x = 360 - (84 + 20 + 96) = 360 - 200 = 160$
 $m\angle l = \frac{1}{2}(160 - 20) = \frac{1}{2}(140) = \frac{\cancel{2} \cdot 70}{\cancel{2}} = 70$



10. $x = 64$
 $y = 116$
 $m\angle DFC = 66$
 $m\widehat{DC} = 180 - 112 = 68$
 $m\angle DFC = \frac{1}{2}(x + m\widehat{DC}) = \frac{1}{2}(64 + 68) = \frac{1}{2}(132) = \frac{\cancel{2} \cdot 66}{\cancel{2}} = 66$
 $24 = \frac{1}{2}(112 - x)$
 $48 = 112 - x$
 $-64 = -x$
 $64 = x$
 $y = 180 - x = 180 - 64 = 116$

—Continued—

In problems 11-14, determine which statements are true and which are false.

11. A secant of a circle has two points of tangency on the circle. false
12. If \overleftrightarrow{AB} and \overleftrightarrow{AC} are tangent to circle Q, then $AB = AC$. false
13. For a given circle, the distance between a point of tangency and the center of the circle equals the radius. true
14. If the intersection of a line and a circle consists of two points, the line is a tangent of the circle. false