Quiz Form A Name ____________________

Unit VI - Circles

Part C - Line and Segment Relationships

Lesson 1 - ***Theorem 73 - "If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects that chord."***

Lesson 2 - ***Theorem 74 -"If a diameter of a circle bisects a chord of the circle which is not a diameter of the circle, then that diameter is perpendicular to that chord."***

Theorem 75 - "If a chord of a circle is a perpendicular bisector of another chord of that circle, then the original chord must be a diameter of the circle."

Lesson 3 - ***Theorem 76 -"If two chords intersect within a circle, then the product of the lengths of the segments of one chord, is equal to the product of the lengths of the segments of the other chord."***

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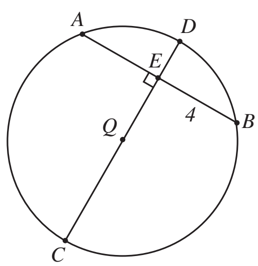
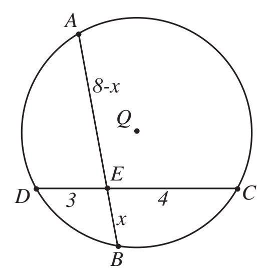
1. Find AE in ⊙Q. AE = <u>4</u>



AE = 4 by Theorem 73
Since $\overline{CD} \perp \overline{AB}$,
$\overline{CD}$ bisects $\overline{AB}$

2. Find AE and BE in ⊙Q. AE = <u>6 or 2</u>
BE = <u>2 or 6 respectively</u>



$AE \cdot BE = DE \cdot EC$

$(8 - x)x = 3 \cdot 4$

$8x - x^2 = 12$

$0 = x^2 - 8x + 12$

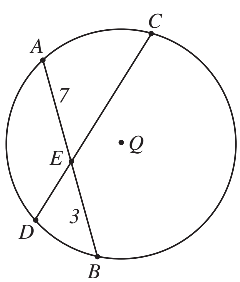
$0 = (x - 6)(x - 2)$

$0 = x - 6 \text{ or } 0 = x - 2$

$6 = x \text{ or } 2 = x$

If x = 6, then 8 - x = 2.
If x = 2, then 8 - x = 6.

3. Find CD in ⊙Q. CD = <u>$\frac{31}{2}$</u>
Given: CE = $2x + 7$
DE = $x - 2$



$AE \cdot BE = DE \cdot EC$

$7 \cdot 3 = (x - 2)(2x + 7)$

$21 = 2x^2 + 3x - 14$

$0 = 2x^2 + 3x - 35$

$0 = (2x - 7)(x + 5)$

$0 = 2x - 7 \text{ or } 0 = x + 5$

$\frac{7}{2} = x \text{ or } -5 = x$

(x cannot be negative)

$CD = DE + CE$

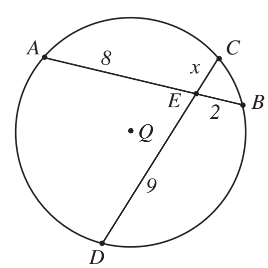
$= (x - 2) + (2x + 7)$

$= \left(\frac{7}{2} - 2\right) + \left(2\left(\frac{7}{2}\right) + 7\right)$

$= \left(\frac{7}{2} - \frac{4}{2}\right) + \left(\frac{14}{2} + \frac{14}{2}\right)$

$= \frac{3}{2} + \frac{28}{2} = \frac{31}{2}$

4. Find CD in ⊙Q. CD = <u>$\frac{97}{9}$</u>



$AE \cdot BE = DE \cdot EC$

$8 \cdot 2 = 9 \cdot x$

$16 = 9x$

$\frac{16}{9} = x$

$CD = CE + DE$

$= x + 9$

$= \frac{16}{9} + 9$

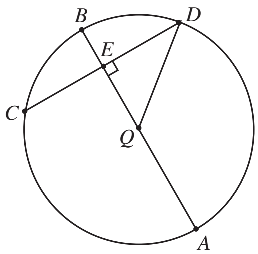
$= \frac{16}{9} + \frac{81}{9}$

$= \frac{97}{9}$

5. Find EQ in ⊙Q, if CD = 10 and DQ = 9.

EQ = $2\sqrt{14}$

DE = 5 from Theorem 73 and $\overline{AB} \perp \overline{CD}$, So $\overline{AB}$ bisects $\overline{CD}$

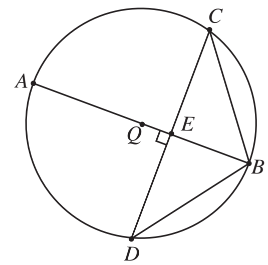


$$(DQ)^2 = (DE)^2 + (EQ)^2$$
$$(9)^2 = (5)^2 + (EQ)^2$$
$$81 = 25 + (EQ)^2$$
$$56 = (EQ)^2$$
$$\pm\sqrt{56} = EQ \quad (EQ \text{ cannot be negative})$$
$$\sqrt{4 \cdot 14} = EQ$$
$$2\sqrt{14} = EQ$$

6. Find m$\widehat{CB}$ in ⊙Q, if m$\widehat{CD}$ = 96°

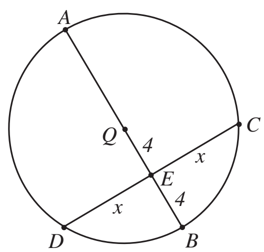
m$\widehat{CB}$ = 48^o

m$\widehat{CB}$ = 48° from Corollary 73a and $\overline{AB} \perp \overline{CD}$, So $\overline{AB}$ bisects $\widehat{CD}$



7. Find DC in ⊙Q.

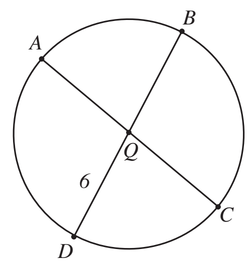
DC = $4\sqrt{3}$



$$AE \cdot BE = DE \cdot EC$$
$$12 \cdot 4 = x \cdot x$$
$$48 = x^2$$
$$\pm\sqrt{48} = x$$
(x cannot be negative)
$$\sqrt{16 \cdot 3} = x$$
$$4\sqrt{3} = x$$

8. Find BD and AC in ⊙Q.

BD = 12
AC = 12



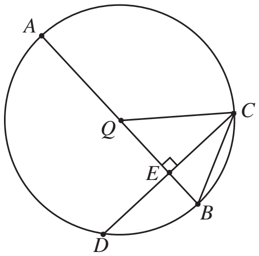
$AQ \cdot QC = BQ \cdot QD$ (call the radius "r")
$$r \cdot r = r \cdot 6$$
$$\frac{\not{r} \cdot r}{\not{r}} = \frac{\not{r} \cdot 6}{\not{r}}$$
$$r = 6$$

$BD = BQ + DQ$	$AC = AQ + CQ$
$BD = 6 + 6$	$AC = 6 + 6$
$BD = 12$	$AC = 12$

$\overline{AC}$ and $\overline{BD}$ are diameters

9. Find mCB in ⊙Q, given that CD = 16, AQ = 9 and EQ = 5

CD = $4\sqrt{5}$

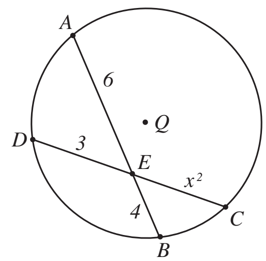


Theorem 73 - $\overline{AB}$ bisects $\overline{CD}$, So CE = 8

$$(CB)^2 = (CE)^2 + (BE)^2$$
$$(CB)^2 = (8)^2 + (9-5)^2$$
$$(CB)^2 = 64 + (4)^2$$
$$(CB)^2 = 64 + 16$$
$$(CB)^2 = 80$$
$$CB = \pm\sqrt{80}$$
(CB cannot be negative)
$$CB = \sqrt{16 \cdot 5}$$
$$CB = 4\sqrt{5}$$

10. Find CE in ⊙Q.

CE = 8



$$AE \cdot BE = DE \cdot EC$$
$$6 \cdot 4 = 3 \cdot x^2$$
$$24 = 3x^2$$
$$8 = x^2$$
$$\pm\sqrt{8} = x$$
(x cannot be negative)
$$\sqrt{4 \cdot 2} = x$$
$$2\sqrt{2} = x$$

$$CE = x^2$$
$$= \left(2\sqrt{2}\right)^2$$
$$= 8$$

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Quiz Form B Name ______________________

Unit VI - Circles

Part C - Line and Segment Relationships

Lesson 1 - ***Theorem 73 - "If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects that chord."***

Lesson 2 - ***Theorem 74 -"If a diameter of a circle bisects a chord of the circle which is not a diameter of the circle, then that diameter is perpendicular to that chord."***

Theorem 75 - "If a chord of a circle is a perpendicular bisector of another chord of that circle, then the original chord must be a diameter of the circle."

Lesson 3 - ***Theorem 76 -"If two chords intersect within a circle, then the product of the lengths of the segments of one chord, is equal to the product of the lengths of the segments of the other chord."***

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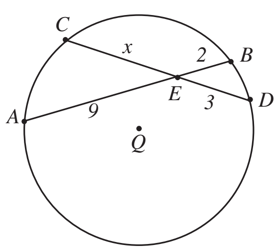
1. Find CE in ⊙Q. CE = $\underline{\ 6\ }$



$AE \cdot BE = DE \cdot EC$

$9 \cdot 2 = 3 \cdot x$

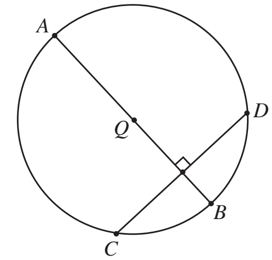
$18 = 3x$

$\frac{18}{3} = x$

$\frac{\not{3} \cdot 6}{\not{3}} = x$

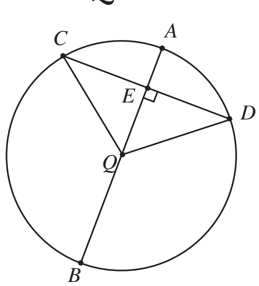
$6 = x$

2. Find $m\overset{\frown}{BD}$ in ⊙Q, if $m\overset{\frown}{CD} = 110$ $m\overset{\frown}{BD} = \underline{\ 55\ }$



mBD = 55 by Corollary 73a
$\overline{AB} \perp \overline{CD}$, *So* $\overline{AB}$ *bisects* $\overset{\frown}{CD}$.

3. Find CD in ⊙Q, if QD = 10 and QE = 4 CD = $\underline{\ 4\sqrt{21}\ }$



$(QD)^2 = (QE)^2 + (DE)^2$

$10^2 = 4^2 + (DE)^2$

$100 = 16 + (DE)^2$

$84 = (DE)^2$

$\pm\sqrt{84} = DE$

(*DE cannot be negative*)

$\sqrt{84} = DE$

$\sqrt{4 \cdot 21} = DE$

$2\sqrt{21} = DE$

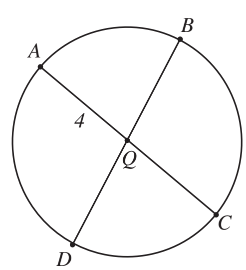
$CD = CE + DE$

$CE = DE$, *by Theorem 73, so,*

$CD = 2\sqrt{21} + 2\sqrt{21}$

$CD = 4\sqrt{21}$

4. If AQ = 4, find BD in ⊙Q. BD = $\underline{\ 8\ }$



$AQ \cdot QC = BQ \cdot QD$

$4 \cdot r = r \cdot r$

$\frac{4 \cdot \not{r}}{\not{r}} = \frac{\not{r} \cdot r}{\not{r}}$

$4 = r$

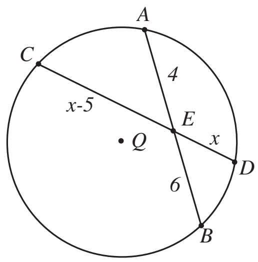
$BD = r + r$

$BD = 4 + 4$

$BD = 8$

($\overline{AC}$ *and* $\overline{BD}$ *are diameters*)

5. Find CD in ⊙Q. CD = 11

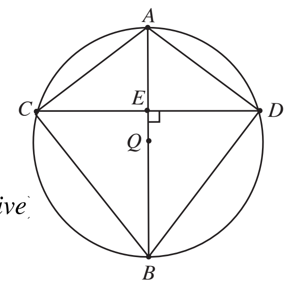


$AE \cdot BE = DE \cdot EC$
$4 \cdot 6 = x \cdot (x - 5)$
$24 = x^2 - 5x$
$0 = x^2 - 5x - 24$
$0 = (x - 8)(x + 3)$
$0 = x - 8 \text{ or } 0 = x + 3$
$x = 8 \text{ or } x - 3$ (*x cannot be negative*)

$CD = DE + EC$
$= x + (x - 5)$
$= 8 + (8 - 5)$
$= 8 + 3$
$= 11$

6. Find BD in ⊙Q, if CD = 10, AE = 2, and QB = 8. BD = $\sqrt{227}$

△ADB is a right triangle since △ADB is inscribe in a semicircle (Corollary 67a)

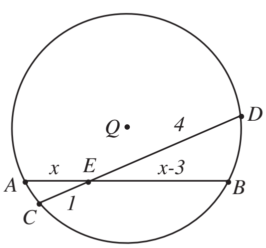


Note that: ED = 5 by Theorem 73. $\overline{AB} \perp \overline{CD}$. So, $\overline{AB}$ bisects $\overline{CD}$

$(AD)^2 = (AE)^2 + (ED)^2$
$(AD)^2 = (2)^2 + (5)^2$
$(AD)^2 = 4 + 25$
$(AD)^2 = 29$
$AD = \pm\sqrt{29}$ (*AD cannot be negative*)

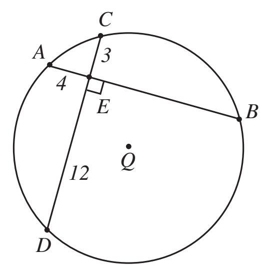
$(AB)^2 = (AD)^2 + (BD)^2$
$(AQ + QB)^2 = (AD)^2 + (BD)^2$
$(8 + 8)^2 = \left(\sqrt{29}\right)^2 + (BD)^2$
$256 = 29 + (BD)^2$
$227 = (BD)^2$
$\pm\sqrt{227} = BD$ (*cannot be negative*)

7. Find AE in ⊙Q. AE = 4



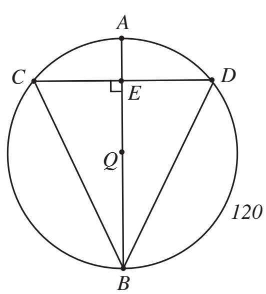
$AE \cdot BE = DE \cdot EC$
$x(x - 3) = 4 \cdot 1$
$x^2 - 3x = 4$
$x^2 - 3x - 4 = 0$
$(x - 4)(x + 1) = 0$
$x - 4 = 0 \text{ or } x + 1 = 0$
$x = 4 \text{ or } x = -1$
(*x cannot be negative*)
$x = 4$

8. Find BE in ⊙Q. BE = 9



$AE \cdot BE = DE \cdot EC$
$4 \cdot BE = 12 \cdot 3$
$4 \cdot BE = 36$
$4BE = 36$
$BE = \frac{36}{4}$
$BE = \frac{\cancel{4} \cdot 9}{\cancel{4}}$
$BE = 9$

9. Find m$\overset{\frown}{CD}$ in ⊙Q. m$\overset{\frown}{CD}$ = 120



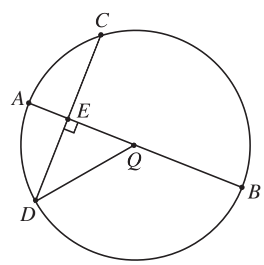
$m\overset{\frown}{AD} = 180 - m\overset{\frown}{BD}$
$m\overset{\frown}{AD} = 180 - 120$
$m\overset{\frown}{AD} = 60$

AB bisects $\overset{\frown}{CD}$
So, $m\overset{\frown}{CD} = 2 \cdot m\overset{\frown}{AD}$

$m\overset{\frown}{CD} = 2 \cdot 60$
$m\overset{\frown}{CD} = 120$

10. Find DQ in ⊙Q, if. QE = 2 and CE = 4. CE = $2\sqrt{5}$

$\overline{AB}$ bisects $\overline{CD}$ (Theorem 73) So if CE = 4, then DE = 4.



$(DQ)^2 = (QE)^2 + (DE)^2$
$(DQ)^2 = (2)^2 + (4)^2$
$(DQ)^2 = 4 + 16$
$(DQ)^2 = 20$
$DQ = \pm\sqrt{20}$ (*DQ cannot be negative*)
$DQ = \sqrt{20}$
$DQ = \sqrt{4 \cdot 5}$
$DQ = 2\sqrt{5}$

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Quiz Form A Name ______________________

Unit VI - Circles
Part C - Line and Segment Relationships

Lesson 4 - ***Theorem 77 - "If two secant segments are drawn to a circle from a single point outside the circle, the product of the lengths of one secant segment and its external segment, is equal to the product of the lengths of the other secant segment and its external segment."***

Theorem 78 - "If a secant segment and a tangent segment are drawn to a circle, from a single point outside the circle, then the length of that tangent segment is the mean proportional between the length of the secant segment, and the length of its external segment."

Lesson 5 - ***Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line must be tangent to the circle, at that endpoint."***

Lesson 6 - ***Theorem 80 - "If two tangent segments are drawn to a circle from the same point outside the circle, then those tangent segments are congruent."***

Lesson 7 - ***Theorem 81 -"If two chords of a circle are congruent, then their intercepted minor arcs are congruent."***

Theorem 82 - "If two minor arcs of a circle are congruent, then the chords which intercept them are congruent."

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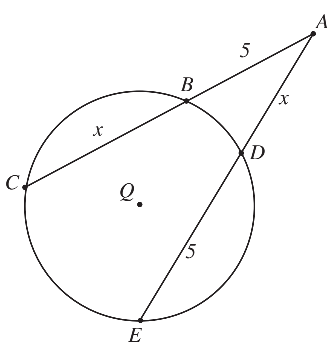
1. Find AC in ⊙Q. AC = 10



$AB \cdot AC = AD \cdot AE$

$5 \cdot (5+x) = x(x+5)$

$25 + 5x = x^2 + 5x$

$0 = x^2 - 25$

$0 = (x+5)(x-5)$

$0 = x+5 \text{ or } 0 = x-5$

$-5 = x \text{ or } 5 = x$

(x cannot be negative)

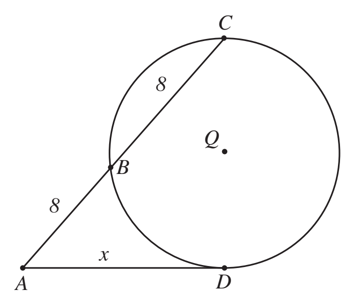
$AC = AB + BC$

$AC = 5 + x$

$AC = 5 + 5$

$AC = 10$

2. Find AD in ⊙Q. AD = $8\sqrt{2}$



$\frac{AC}{AD} = \frac{AD}{AB}$

$\frac{8+8}{x} = \frac{x}{8}$

$\frac{16}{x} = \frac{x}{8}$

$x \cdot x = 16 \cdot 8$

$x^2 = 128$

$x = \pm\sqrt{128}$

(x cannot be negative)

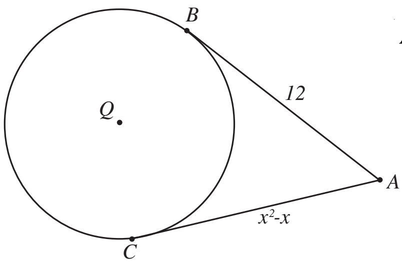
$x = \sqrt{128}$

$x = \sqrt{64 \cdot 2}$

$x = 8\sqrt{2}$

$AD = 8\sqrt{2}$

3. Find x in ⊙Q. $x =$ <u>4</u>



$AB = AC$

$12 = x^2 - x$

$0 = x^2 - x - 12$

$0 = (x-4)(x+3)$

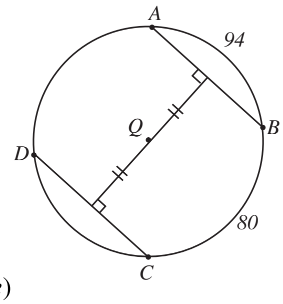
$0 = x - 4$ or $0 = x + 3$

$4 = x$ or $-3 = x$

(x cannot be negative)

$x = 4$

4. Find m$\overset{\frown}{AD}$ in ⊙Q. m$\overset{\frown}{AD}$ = <u>92</u>



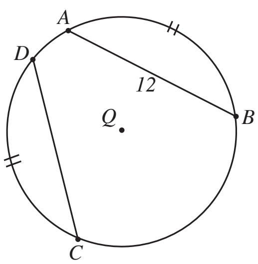
If we draw radii $\overline{QA}$, $\overline{QB}$, $\overline{QC}$, and $\overline{QD}$, we can prove triangles congruent. We can then use Theorem 73 and prove $\overline{AB} \cong \overline{CD}$.

$m\overset{\frown}{AD} = 360 - m\overset{\frown}{AB} - m\overset{\frown}{BC} - m\overset{\frown}{CD}$

$m\overset{\frown}{AD} = 360 - 94 - 80 - 94$

$m\overset{\frown}{AD} = 92$

5. Find CD in ⊙Q. CD = <u>12</u>

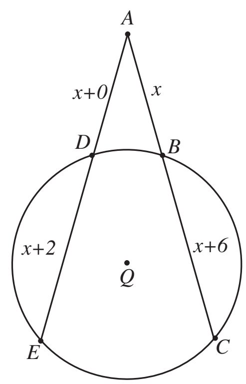


$\overline{CD} \cong \overline{AB}$

$CD = AB$

$CD = 12$

6. Find AC and AE in ⊙Q. AC = <u>12</u> AE = <u>9</u>



$AB \cdot AC = AD \cdot AE$

$x[x + (x+6)] = (x+1)[(x+1) + (x+2)]$

$x(2x+6) = (x+1)(2x+3)$

$2x^2 + 6x = 2x^2 + 5x + 3$

$6x = 5x + 3$

$x = 3$

$AC = x + x + 6$

$AC = (3) + (3) + 6$

$AC = 12$

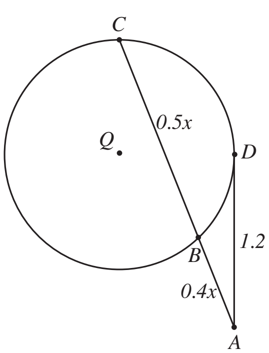
$AE = (x+1) + (x+2)$

$AE = (3+1) + (3+2)$

$AE = 4 + 5$

$AE = 9$

7. Find AB and BC in ⊙Q. AB = <u>0.8</u> BC = <u>1</u>



$\frac{AC}{AD} = \frac{AD}{AB}$

$\frac{0.9x}{1.2} = \frac{1.2}{0.4x}$

$(1.2)(1.2) = (0.9x)(0.4x)$

$1.44 = 0.36x^2$

$\frac{1.44}{0.36} = x^2$

$4 = x^2$

$\pm\sqrt{4} = x$ (x cannot be negative)

$\sqrt{4} = x$

$2 = x$

$AB = (0.4) \cdot x$

$AB = (0.4)2$

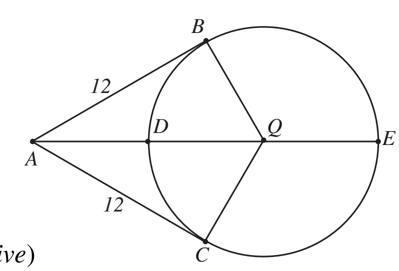
$AB = 0.8$

$BC = (0.5) \cdot x$

$BC = (0.5)2$

$BC = 1$

8. AB and AC are tangents to ⊙Q, and m∠BAC = 42. Find m∠BAE. m∠BAE = <u>21°</u>



$\overline{AE}$ bisects ∠BAC by Corollary 80a

$m\angle BAE = \frac{1}{2} \cdot m\angle BAC$

$= \frac{1}{2} \cdot 42$

$= 21$

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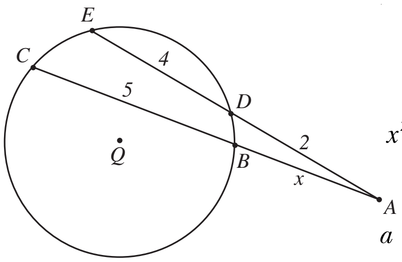
9. Find AB in ⊙Q. AB = $\frac{-5+\sqrt{73}}{2}$ *or 1.772*



$AB \cdot AC = AD \cdot AE$

$x(x+5) = 2 \cdot 6$

$x^2 + 5x = 12$

$x^2 + 5x - 12 = 0$

$a = 1 \quad b = 5 \quad c = -12$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25+48}}{2}$$

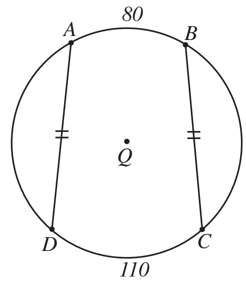
$$x = \frac{-5 \pm \sqrt{73}}{2}$$

$$x = \frac{-5+\sqrt{73}}{2} \text{ or } \frac{-5-\sqrt{73}}{2}$$

(*x cannot be negative*)

$$x = \frac{-5+\sqrt{73}}{2} \text{ or } 1.772$$

10. Find $m\overset{\frown}{BC}$ in ⊙Q. $m\overset{\frown}{BC}$ = *85*



$m\overset{\frown}{BC} = 360 - 80 - 110 - m\overset{\frown}{AD}$

$m\overset{\frown}{BC} = m\overset{\frown}{AD}$

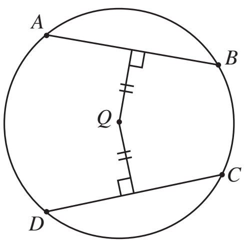
$m\overset{\frown}{BC} = 360 - 80 - 110 - m\overset{\frown}{BC}$

$2m\overset{\frown}{BC} = 170$

$m\overset{\frown}{BC} = \frac{85 \cdot \cancel{2}}{\cancel{2}}$

$m\overset{\frown}{BC} = 85$

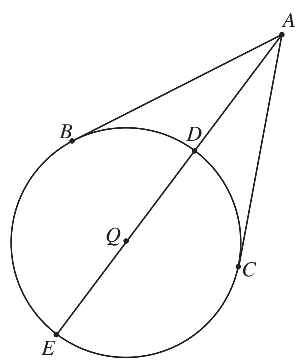
11. If AB = 7, find CD in ⊙Q. CD = *7*



Since $\overline{AB}$ and $\overline{CD}$ are equidistant from point Q, $\overline{AB} \cong \overline{CD}$. CD = 7

(See Unit VI, Part C, Lesson 7, Exercise 3)

12. Find AB and AC in ⊙Q. if AD = 8 and DE = 12 AB = $4\sqrt{10}$ AC = $4\sqrt{10}$



$$\frac{AD}{AB} = \frac{AB}{AE}$$

$$\frac{8}{AB} = \frac{AB}{8+12}$$

$$\frac{8}{AB} = \frac{AB}{20}$$

$(AB)^2 = 160$

$AB = \pm\sqrt{160}$

(*AB cannot be negative*)

$AB = \sqrt{160}$

$AB = \sqrt{16 \cdot 10}$

$AB = 4\sqrt{10}$

$AB = AC$

$4\sqrt{10} = AC$

Quiz Form B Name ____________________

Unit VI - Circles
Part C - Line and Segment Relationships

Lesson 4 - ***Theorem 77 - "If two secant segments are drawn to a circle from a single point outside the circle, the product of the lengths of one secant segment and its external segment, is equal to the product of the lengths of the other secant segment and its external segment."***

Theorem 78 - "If a secant segment and a tangent segment are drawn to a circle, from a single point outside the circle, then the length of that tangent segment is the mean proportional between the length of the secant segment, and the length of its external segment."

Lesson 5 - ***Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line must be tangent to the circle, at that endpoint."***

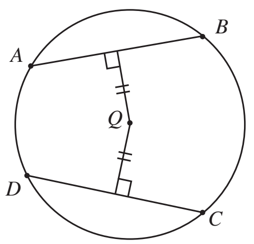
Lesson 6 - ***Theorem 80 - "If two tangent segments are drawn to a circle from the same point outside the circle, then those tangent segments are congruent."***

Lesson 7 - ***Theorem 81 -"If two chords of a circle are congruent, then their intercepted minor arcs are congruent."***

Theorem 82 - "If two minor arcs of a circle are congruent, then the chords which intercept them are congruent."

---

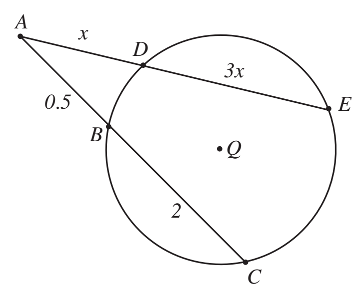
1. In ⊙Q, if AB = 9, why is CD = 9?



Chord $\overline{AB}$ and chord $\overline{CD}$ are the same distance from the center of the circle, point Q. (See Unit VI, Part C, Lesson 7, Exercise 3)

2. Find DE in ⊙Q.

DE = $\frac{15\sqrt{.05}}{2}$



$AB \cdot AC = AD \cdot AE$

$(0.5)(2.5) = (x)(4x)$

$1.25 = 4x^2$

$\frac{1.25}{4} = x^2$

$\pm\sqrt{\frac{1.25}{4}} = x$

(*x cannot be negative*)

$\sqrt{\frac{1.25}{4}} = x$

$\frac{\sqrt{1.25}}{2} = x$

$\frac{\sqrt{25 \cdot .05}}{2} = x$

$\frac{5\sqrt{.05}}{2} = x$

$.5590 \doteq x$

$DE = 3x$

$DE = 3 \cdot \frac{5\sqrt{.05}}{2}$

$DE \doteq 1.6771$

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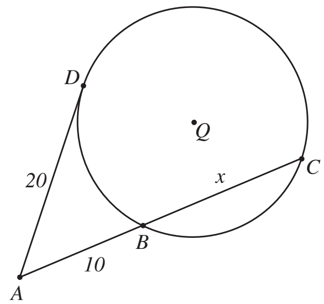
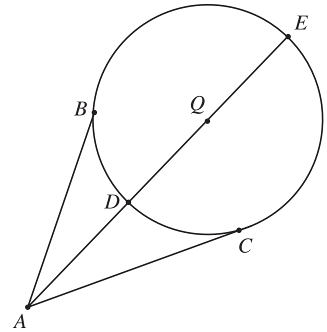
3. Find BC in ⊙Q. BC = *30*



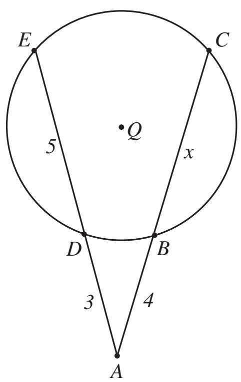
$$\frac{AB}{AD}=\frac{AD}{AC}$$
$$\frac{10}{20}=\frac{20}{10+x}$$
$$(20)(20)=(10)(10+x)$$
$$400=100+10x$$
$$300=10x$$
$$30=x$$
$$BC=x$$
$$BC=30$$

4. If, in ⊙Q, m∠BAC = 52, find m∠CAE. m∠CAE = *26°*

AE bisects ∠BAC
m∠CAE = 26 degrees

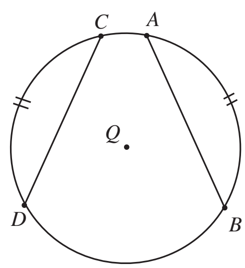


5. Find AC in ⊙Q. AC = *6*

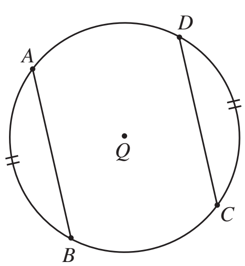


$$AB\cdot AC=AD\cdot AE$$
$$4(4+x)=3(3+5)$$
$$16+4x=9+15$$
$$16+4x=24$$
$$4x=8$$
$$x=2$$
$$AC=AB+BC$$
$$AC=4+x$$
$$AC=4+2$$
$$AC=6$$

6. If, in ⊙Q, AB = 8, why is CD = 8? *If two minor arcs are congruent, then their chords are congruent.*



7. Find AB and CD in ⊙Q, given that DC = $x^2 + 2x$ and AB = $2x^2$ - 8. AB = *24* CD = *24*



If two minor arcs are congruent, then their chords are congruent.

$$\overline{AB}\cong\overline{CD}$$
$$AB=CD$$
$$2x^2-8=x^2+2x$$
$$x^2-2x-8=0$$
$$(x+2)(x-4)=0$$
$$x+2=0 \text{ or } x-4=0$$
$$x=-2 \text{ or } x=4$$

If $x=-2$, *then*

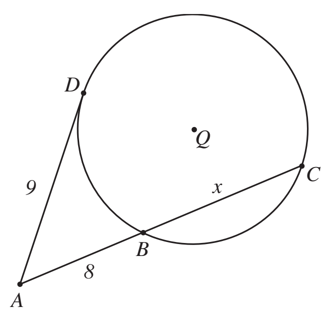
$$AB=2(-2)^2-8 = 2(4)-8 = 0$$
$$CD=(-2)^2+2\cdot -2 = 4-4 = 0$$

AB and CD cannot be zero.

If $x=4$, *then*

$$AB=2(4)^2-8 = 2(16)-8 = 24$$
$$CD=(4)^2+2\cdot 4 = 16+8 = 24$$

8. Find AC in ⊙Q. AC = $\frac{81}{8}$



$$\frac{AB}{AD}=\frac{AD}{AC}$$
$$\frac{8}{9}=\frac{9}{8+x}$$
$$(9)(9)=(8)(8+x)$$
$$81=64+8x$$
$$17=8x$$
$$\frac{17}{8}=x$$
$$AC=AB+BC$$
$$AC=8+x$$
$$AC=8+\frac{17}{8}$$
$$AC=\frac{64}{8}+\frac{17}{8}$$
$$AC=\frac{81}{8}$$

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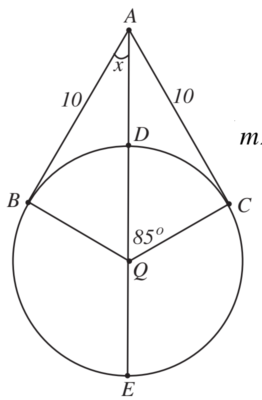
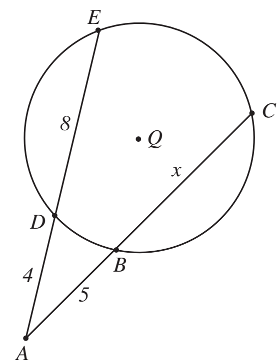
9. If, in ⊙Q, $\overline{AB}$ and $\overline{AC}$ are tangent segments, find *x*. $x =$ 5



$\overline{AE}$ *bisects* $\angle BAC$
$\overline{QC} \perp \overline{AC}$ *(Corollary 68a)*
$\triangle ACQ$ *is a right triangle.*

$$m\angle CAQ + m\angle AQC + m\angle ACQ = 180$$
$$m\angle CAQ + 85 + 90 = 180$$
$$m\angle CAQ = 5$$
$$\angle BAQ \cong \angle CAQ$$
$$x = 5$$

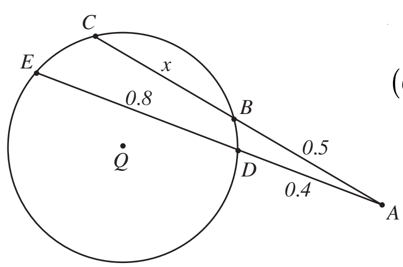
10. Find AC in ⊙Q. AC = $\frac{48}{5}$



$$AB \cdot AC = AD \cdot AE$$
$$5(5+x) = 4(4+8)$$
$$25 + 5x = 16 + 32$$
$$25 + 5x = 48$$
$$5x = 23$$
$$x = \frac{23}{5}$$

$$AC = AB + BC$$
$$AC = 5 + \frac{23}{5}$$
$$AC = \frac{25}{5} + \frac{23}{5}$$
$$AC = \frac{48}{5}$$

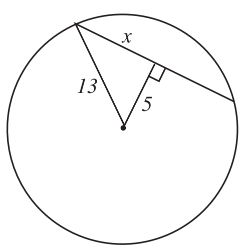
11. Find BC in ⊙Q. BC = 0.46



$$AB \cdot AC = AD \cdot AE$$
$$(0.5)(0.5 + x) = 0.4(0.4 + 0.8)$$
$$0.25 + 0.5x = 0.4(1.2)$$
$$0.25 + 0.5x = 0.48$$
$$0.5x = 0.23$$
$$x = \frac{0.23}{0.5}$$
$$x = 0.46$$

$$BC = x$$
$$BC = 0.46$$

12. A chord is located 5 inches from the center of a circle with radius 13. Find the length of the chord. chord = 24 inches



$$x^2 + 5^2 = 13^2$$
$$x^2 + 25 = 169$$
$$x^2 = 144$$
$$x = \pm 12$$
$$(x \text{ cannot be negative})$$
$$x = 12$$

The length of the chord is 2x
$$2x$$
$$2(12)$$
$$24$$
The length is 24 inches

Quiz Form A Name ______________________________

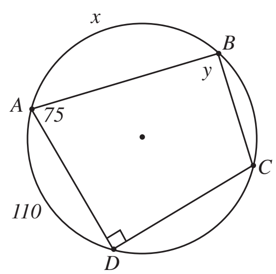
Unit VI - Circles
Part D - Circle Concurrency

Lesson 1 - *Theorem 83 - "If you have a triangle, then that triangle is cyclic."*

Lesson 2 - *Theorem 84 - "If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic."*

---

1. Quadrilateral ABCD is cyclic. Find x and y. $x =$ ___*100*___ $y =$ ___*90*___



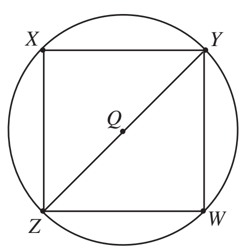
∠ABC and ∠ADC are supplementary. (Corollary 67b)

$$m\angle ABC + m\angle ADC = 180$$
$$y + 90 = 180$$
$$y = 90$$

$$m\overset{\frown}{AB} + m\overset{\frown}{BCD} + m\overset{\frown}{DA} = 360$$
$$x + 150 + 110 = 360$$
$$x + 260 = 360$$
$$x = 100$$

$$m\angle BAD = \frac{1}{2}\cdot m\overset{\frown}{BD} \text{ (Theorem 67)}$$
$$75 = \frac{1}{2}\cdot m\overset{\frown}{BD}$$
$$150 = m\overset{\frown}{BD} \text{ (or } m\overset{\frown}{BCD}\text{)}$$

2. Quadrilateral (Kite) ABCD is cyclic. Find $m\overset{\frown}{AB}$. $\overset{\frown}{AB} =$ ___*46*___

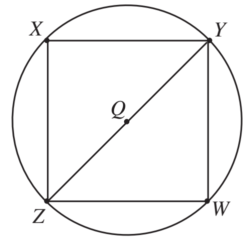


Quadrilateral ABCD is a kite, so $\overline{CD} \cong \overline{CB}$, $\overset{\frown}{BC} \cong \overset{\frown}{DC}$ (Theorem 81), and $m\overset{\frown}{BC} = 134$.

$$m\overset{\frown}{AB} + m\overset{\frown}{BC} = 180 \text{ (}\overset{\frown}{ABC} \text{ is a semicircle)}$$
$$m\overset{\frown}{AB} + 134 = 180$$
$$m\overset{\frown}{AB} = 46$$

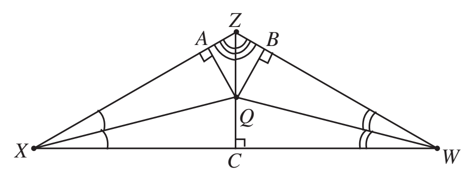
3. Given: Quadrilateral XYWZ is cyclic.
$\overline{ZY}$ is a diameter of ⊙Q.
$\widehat{XY} \cong \widehat{WZ}$

Prove: ∠XYZ ≅ ∠WZY



STATEMENT	REASON
1. Quadrilateral XYZW is cyclic	*1. Given*
2. $\overline{ZY}$ is a diameter of ⊙Q	*2. Given*
3. $\widehat{XY} \cong \widehat{WZ}$	*3. Given*
4. ∠ZXY is a right angle	*4. Corollary 67a - If you have an angle inscribed in a semicircle, then that angle must be a right angle.*
5. △ZXY is a right triangle	*5. Definition of Right Triangle*
6. ∠YWZ is a right angle	*6. Corollary 67a*
7. △YWZ is a right triangle	*7. Definition of Right Triangle*
8. $\overline{XY} \cong \overline{WZ}$	*8. Theorem 82 - If two minor arcs are congruent, then their chords are congruent.*
9. $\overline{YZ} \cong \overline{ZY}$	*9. Reflexive Property for Congruent Line Segments*
10. △ZXY ≅ △YWZ	*10. Postulate Corollary 13c - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two right triangles are congruent. (HL)*
11. ∠XYZ ≅ ∠WZY	*11. C.P.C.T.C.*

4. The angle bisectors of the angles of △XYZ meet at point Q. QX = 75 and QC = 20. Find QB. Explain your answer.



The bisectors of the angles of a triangle are concurrent at a point which is equidistant from the sides of the triangle. (Corollary 83b) Therefore, QC = QB. So, QB = 20.

QB = *20*

Complete the following statements by choosing “sometimes”, “always”, or “never”.

5. Rectangles are *always* cyclic quadrilaterals.

6. Irregular quadrilaterals are *sometimes* cyclic.

7. Regular polygons are *always* cyclic.

8. A kite is *sometimes* a cyclic quadrilateral.

9. Opposite angles of a cyclic quadrilateral *always* add up to 180 degrees.

10. Isosceles trapezoids are *always* cyclic quadrilaterals.

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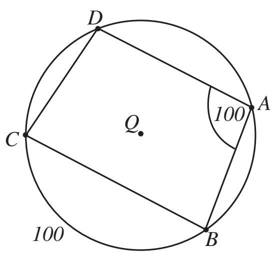
Quiz Form B Name ____________________

Unit VI - Circles
Part D - Circle Concurrency
Lesson 1 - *Theorem 83 - "If you have a triangle, then that triangle is cyclic."*

Lesson 2 - *Theorem 84 - "If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic."*

---

1. Quadrilateral ABCD is cyclic with the measures shown below. Find $m\overset{\frown}{CD}$. $\overset{\frown}{CD}$ = *90*



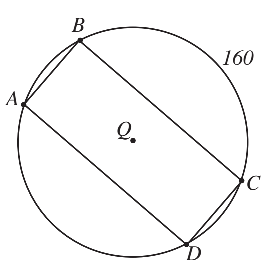
∠BAD and ∠BCD are supplementary. (Corollary 67b)

$m\angle BAD + m\angle BCD = 180$
$100 + m\angle BCD = 180$
$m\angle BCD = 80$

$m\angle BCD = \frac{1}{2} \cdot m\overset{\frown}{BD}$ (Theorem 67)
$80 = \frac{1}{2} \cdot m\overset{\frown}{BD}$
$160 = m\overset{\frown}{BD}$ (or $m\overset{\frown}{BAD}$)

$m\overset{\frown}{CD} + m\overset{\frown}{BAD} + m\overset{\frown}{BC} = 360$
$m\overset{\frown}{CD} + 160 + 110 = 360$
$m\overset{\frown}{CD} + 270 = 360$
$m\overset{\frown}{CD} = 90$

2. Quadrilateral ABCD is a cyclic rectangle, with the measures shown below. Find $m\overset{\frown}{CD}$. $\overset{\frown}{CD}$ = *20*

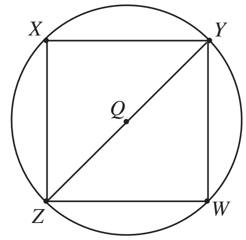


∠BCD is a right angle, so $\overline{BD}$ must be a diameter.
$\overset{\frown}{BCD}$ is a semicircle

$m\overset{\frown}{BC} + m\overset{\frown}{CD} = m\overset{\frown}{BCD}$
$160 + m\overset{\frown}{CD} = 180$
$m\overset{\frown}{CD} = 20$

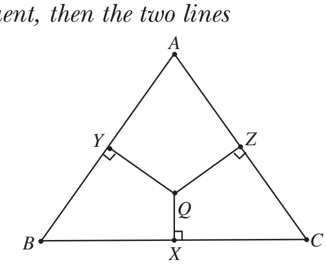
3. Given: Quadrilateral XYWZ is cyclic, with diameter $\overline{ZY}$.
$\overset{\frown}{XY} \cong \overset{\frown}{WZ}$

Prove: $\overline{XY} \parallel \overline{WZ}$



STATEMENT	REASON
1. Quadrilateral XYZW is cyclic	*1. Given*
2. $\overset{\frown}{XY} \cong \overset{\frown}{WZ}$	*2. Given*
3. $\overline{XY} \cong \overline{ZW}$	*3. Theorem 82 - If two minor arcs are congruent, then their chords are congruent.*
4. $\angle ZXY$ *is a right angle*	*4. Corollary 67a - If you have an angle inscribed in a semicircle, then that angle must be a right angle.*
5. $\triangle ZXY$ *is a right triangle*	*5. Definition of Right Triangle*
6. $\angle YWZ$ *is a right angle*	*6. Corollary 67a*
7. $\triangle YWZ$ *is a right triangle*	*7. Definition of Right Triangle*
8. $\overline{YZ} \cong \overline{ZY}$	*8. Reflexive Property for Congruent Line Segments*
9. $\triangle ZXY \cong \triangle YWZ$	*9. Postulate Corollary 13c - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two right triangles are congruent. (HL)*
10. $\angle XYZ \cong \angle WZY$	*10. C.P.C.T.C.*
11. $\overline{XY} \parallel \overline{WZ}$	*11. Corollary 20a - If two lines are cut by a transversal so that alternate interior angles are congruent, then the two lines are parallel.*

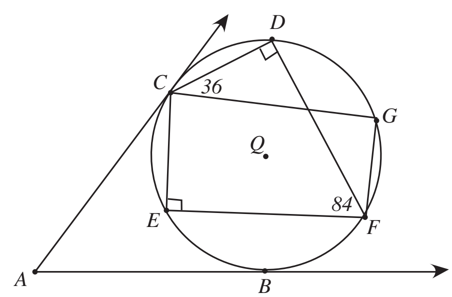
4. The perpendicular bisectors of the sides of $\triangle ABC$ meet at point Q. QX = 2 and QA = 7. Find QC. Explain your answer.



The perpendicular bisectors of the sides of a triangle are concurrent at a point which is equidistant from the vertices of the triangle. (Corollary 83a) Therefore, QA = QC. So, QC = 7

QC = *7*

Use the figure to the right for problems 5 - 10.



5. Find $m\angle CGF$. *90 (∠CDF and ∠CGF intercept the same arc)*

6. Find $m\overset{\frown}{ED}$. *168 (m∠DFE = 1/2 • m$\overset{\frown}{ED}$)*

7. Find $m\overset{\frown}{CBF}$. *180 ($\overset{\frown}{CBF}$ is a semicircle)*

8. Find $m\angle GFD$. *36 (∠GFD and ∠GCD intercept the same arc)*

9. Find $m\overset{\frown}{DG}$. *72 (m∠GCD = = 1/2 • m$\overset{\frown}{DG}$)*

10. Find $m\angle ECG$. *150 (360 - 90 - 84 - 36 = 150 using quadrilateral CGFE)*

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Name ____________________

Class ________ Date ________ Score ________

Unit VI - Circles

Match each term in Column I with a phrase in Column II which best fits that term.

___*e*___	*(A-1)* **1.** Tangent	**a)** A line that intersects a circle in two points
___*g*___	*(A-2)* **2.** Central Angle	**b)** A line segment whose endpoints are the center of a circle and a point on the circle.
___*k*___	*(A-2)* **3.** Major Arc	**c)** A line segment whose endpoints are on the circle.
___*l*___	*(A-2)* **4.** Intercepted Arc	**d)** The set of all points of a circle determined by the endpoints of a diameter.
___*c*___	*(A-1)* **5.** Chord	**e)** A line that intersects a circle in exactly one point.
___*h*___	*(A-1)* **6.** Diameter	**f)** The set of points in a plane such that all points are the same distance from a given point.
___*i*___	*(A-2)* **7.** Minor Arc	**g)** An angle whose vertex is the center of a circle.
___*a*___	*(A-1)* **8.** Secant	**h)** A line segment whose endpoints are on the circle and that contains the center of the circle.
___*f*___	*(A-1)* **9.** Circle	**i)** The set of all points of a circle on or inside a central angle.
___*j*___	*(A-3)* **10.** Inscribed Polygon	**j)** A polygon whose sides are chords of a circle.
___*d*___	*(A-2)* **11.** Semicircle	**k)** The set of all points of a circle on or outside a central angle.
___*b*___	*(A-3)* **12.** Radius	**l)** The set of all points of a circle determined by the points of intersection of the circle and the sides of an angle of the circle.

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Name ______________________________

Determine whether each of the following is always, sometimes, or never true.

never (C-7) **13.** Congruent chords of different circles intercept congruent arcs.

always (B-2) **14.** An angle inscribed in a semicircle is a right angle.

always (A-1) **15.** Two circles are congruent if their radii are congruent.

never (A-3) **16.** Two externally tangent circles have two common tangents.

never (A-1) **17.** A radius is a segment that joins two points on a circle.

sometimes (A-3) **18.** A polygon inscribed in a circle is a regular polygon.

always (A-1) **19.** A secant is a line that lies in the plane of a circle, and contains a chord of the circle.

always (B-2) **20.** The opposite angles of an inscribed quadrilateral are supplementary.

always (Postulate 8 - p198) **21.** If point X is on $\overset{\frown}{AB}$, then $m\overset{\frown}{AX} + m\overset{\frown}{XB} = m\overset{\frown}{AXB}$.

always (A-3) **22.** The common tangent segments of two circles of unequal radii are congruent.

sometimes (A-3) **23.** Tangent segments from an external point to two different circles are congruent.

sometimes (D-2) **24.** Cyclic quadrilaterals are congruent.

never (A-3) **25.** If two circles are internally tangent, then the circles have three common tangents.

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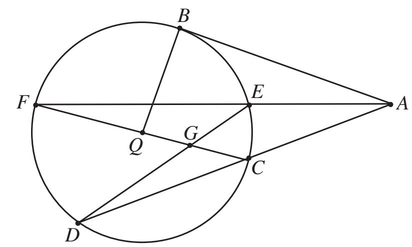


Use the given figure to answer problems 26 to 35.
(Note: $\overline{AB}$ is tangent to Q at point B)

(B-2)
26. If m$\overset{\frown}{DF}$ = 96, find m∠DEF.

m∠DEF = *48*

$$m\angle DEF = \frac{1}{2}\cdot m\overset{\frown}{DF}\ (Theorem\ 67)$$
$$= \frac{1}{2}\cdot 96$$
$$= \frac{1\cdot \cancel{2}\cdot 48}{\cancel{2}}$$
$$= 48$$

(B-4)
27. If m$\overset{\frown}{CD}$ = 62 and m∠EGF = 110, find m$\overset{\frown}{EF}$.

m$\overset{\frown}{EF}$ = *158*

$$m\angle EGF = \frac{1}{2}\cdot\left(m\overset{\frown}{CD} + m\overset{\frown}{EF}\right)(Theorem\ 69)$$
$$110 = \frac{1}{2}\cdot\left(62 + m\overset{\frown}{EF}\right)$$
$$220 = 62 + m\overset{\frown}{EF}$$
$$158 = m\overset{\frown}{EF}$$

(B-4)
28. If m$\overset{\frown}{DF}$ = 96 and m$\overset{\frown}{CE}$ = 40, find m∠FAD.

m∠FAD = *28*

$$m\angle FAD = \frac{1}{2}\cdot\left(m\overset{\frown}{DF} - m\overset{\frown}{CE}\right)(Theorem\ 70)$$
$$= \frac{1}{2}\cdot(96 - 40)$$
$$= \frac{1}{2}\cdot 56$$
$$= \frac{1\cdot \cancel{2}\cdot 28}{\cancel{2}}$$
$$= 28$$

(B-5)
29. If m$\overset{\frown}{BFD}$ = 170 and m$\overset{\frown}{BC}$ = 110, find m∠BAD.

m∠BAD = *30*

$$m\angle BAD = \frac{1}{2}\cdot\left(m\overset{\frown}{BFD} - m\overset{\frown}{BC}\right)(Theorem\ 71)$$
$$= \frac{1}{2}\cdot(170 - 110)$$
$$= \frac{1}{2}\cdot 60$$
$$= \frac{1\cdot \cancel{2}\cdot 30}{\cancel{2}}$$
$$= 30$$

(B-3)
30. Find m∠ABQ.

m∠ABQ = *90*

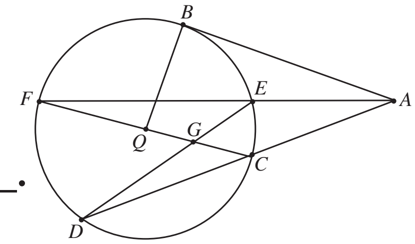
$m\angle ABQ = 90\ (Corollary\ 68a)$

(B-4)
31. If m∠ADE = 26, find m$\overset{\frown}{CE}$.

m∠BAD = *52*

$$m\angle ADE = \frac{1}{2}\cdot m\overset{\frown}{CE}\ (Theorem\ 67)$$
$$26 = \frac{1}{2}\cdot m\overset{\frown}{CE}$$
$$52 = m\overset{\frown}{CE}$$

Name ______________________________



(B-2)
32. If m∠ADE = 26, find m∠AFC.

m∠AFC = *26*

∠ADE ≅ ∠AFC (Corollary 67c)

$m\angle ADE = \frac{1}{2}\cdot m\widehat{CE}$

$26 = \frac{1}{2}\cdot m\widehat{CE}$

$52 = m\widehat{CE}$

$m\angle AFC = \frac{1}{2}\cdot m\widehat{CE}$

$= \frac{1}{2}\cdot 52$

$= \frac{1\cdot \cancel{2}\cdot 26}{\cancel{2}}$

$= 26$

(B-2)
33. ∠DCF ≅ *∠FED*.

∠FED (Corollary 67)

(B-5)
34. If m∠FAB = 18 and $m\widehat{BE}$ = 80, find $m\widehat{BF}$.

$m\widehat{BF}$ = *116*

$m\angle FAB = \frac{1}{2}\cdot\left(m\widehat{BF} - m\widehat{BE}\right)$ *(Theorem 71)*

$18 = \frac{1}{2}\cdot\left(m\widehat{BF} - 80\right)$

$36 = m\widehat{BF} - 80$

$116 = m\widehat{BF}$

(A-2)
35. If m∠BQF = 90, find $m\widehat{BF}$.

$m\widehat{BF}$ = *90*

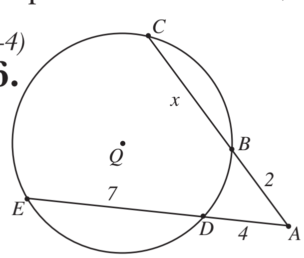
$m\widehat{BF} = m\angle BQF$

The measure of a central angle is the same as the measure of its intercepted arc.

$m\widehat{BF} = 90$

For problems 36 to 41, find the value of x.

(C-4)
36.



x = *20*

$(AC)(AB) = (AE)(AD)$ *Theorem 77*

$(2+x)(2) = (4+7)(4)$

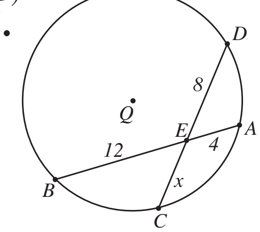
$4 + 2x = 16 + 28$

$2x = 16 + 28 - 4$

$2x = 40$

$x = 20$

(C-3)
37.



x = *6*

$(AE)(EB) = (CE)(ED)$ *Theorem 76*

$(4)(12) = (x)(8)$

$48 = 8x$

$6 = x$

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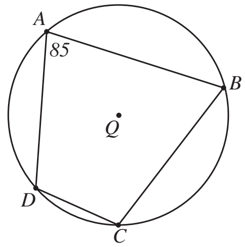
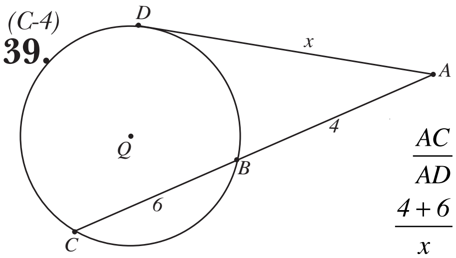
(B-2)
38. Find m∠C m∠C = 95



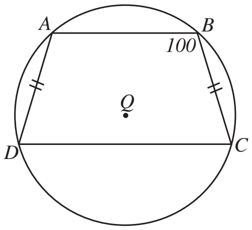
$m\angle A + m\angle C = 180$ (Corollary 67b)
$85 + m\angle C = 180$
$m\angle C = 95$

(C-4)
39. $x = 2\sqrt{10}$



$\frac{AC}{AD} = \frac{AD}{AB}$ (Theorem 78)
$\frac{4+6}{x} = \frac{x}{4}$
$x \cdot x = (4)(4+6)$
$x^2 = 4 \cdot 10$
$x = \pm\sqrt{4 \cdot 10}$
(x cannot be negative)
$x = \sqrt{4} \cdot \sqrt{10}$
$x = 2\sqrt{10}$

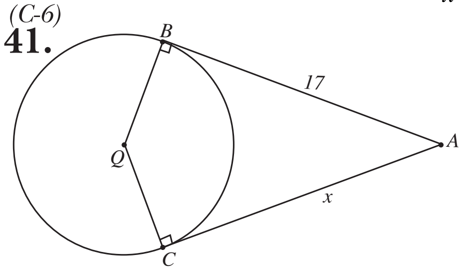
(B-2)
40. Find m∠C m∠D = 80



$m\angle D + m\angle B = 180$
A
is cyclic and Corollary 67b

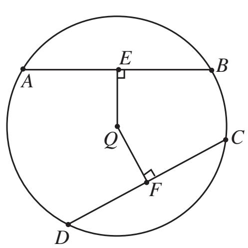
$m\angle D + 100 = 180$
$= 80$

(C-6)
41. $x = 17$



$x = 17$; $\overline{AB} \cong \overline{AC}$
(Theorem 80)

(C-1)
42. Find QE QE = 3



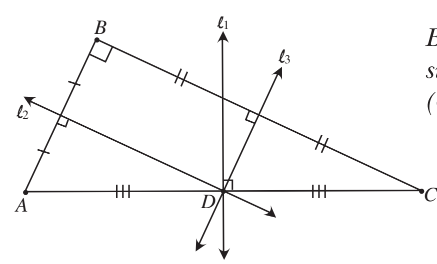
Given: $\overline{AB} \cong \overline{CD}$, DC = 8, the radius of ⊙Q is 5.

If DC = 8, then FC = 4
(Theorem 73)

$(QF)^2 + (FC)^2 = (QC)^2$
$(QF)^2 + (4)^2 = (5)^2$
$(QF)^2 + 16 = 25$
$(QF)^2 = 9$
$QF = \pm\sqrt{9}$
(QF cannot be negative)
$QF = \sqrt{9}$
$QF = 3$

so, QE = 3. Congruent Chords in the same circle are equidistant from the center of the circle. (See Unit VI, Part C, Lesson 7, Exercise 3)

(D-1)
43. Find BD. BD = 10



Given: ℓ_1 and ℓ_3 are perpendicular bisectors of the sides of △ABC. AC = 20.

BD = 10,
since AD = CD = BD
(Corollary 83a)

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Unit VI - Circles

Match each term in Column I with a phrase in Column II.

e	(A-3) **1.** Common Internal Tangent	**a)** Circles in the same plane with the same center.
f	(A-3) **2.** Tangent Circles	**b)** A line segment on a tangent of a circle one of the endpoints of which is the point of tangency.
h	(A-2) **3.** Inscribed Angle	**c)** Measure of central angle AQB of ⊙Q
k	(A-3) **4.** Circumscribed Polygon	**d)** A point on a tangent line of a circle where the line is tangent to the circle.
j	(A-3) **5.** Externally Tangent Circles	**e)** Any common tangent which intersects the line segment joining the centers of two circles
c	(A-2) **6.** Measure of $\widehat{AB}$ of ⊙Q	**f)** Two circles that intersect in exactly one point.
i	(D-2) **7.** Cyclic Quadrilateral	**g)** A common tangent of two circles that does not intersect the line segment joining the centers of the two circles.
a	(A-3) **8.** Concentric Circles	**h)** An angle whose vertex is on a circle and whose sides intersect the circle in two other points.
b	(A-1) **9.** Tangent Segment	**i)** A quadrilateral around which a circle can be drawn to pass through all its vertices.
l	(D-1) **10.** Bisectors of the angles of a Triangle.	**j)** Two circles in the same plane that touch at only one point neither of which is inside the other.
d	(A-1) **11.** Point of Tangency	**k)** A polygon whose sides are tangent segments of a circle.
g	(A-3) **12.** Common External Tangent	**l)** Lines which are concurrent at a point equidistant from the three sides of a triangle.

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Determine whether each of the following is always, sometimes or never true.

always 13. *(A-1)* The longest chord of a circle is a diameter.

never 14. *(B-2)* An angle inscribed in a semicircle has a measure of 180°.

sometimes 15. *(C-1)* A line perpendicular to a chord of a circle bisects the chord.

sometimes 16. *(C-7)* If the measure of an arc in one circle is equal to the measure of an arc in another circle, the chords for these arcs are congruent.

never 17. *(B-2)* If a quadrilateral is inscribed in a circle, then its opposite angles are complimentary.

always 18. *(C-5)* If a line is perpendicular to a radius of a circle at a point on the circle, then the line is tangent to the circle.

sometimes 19. *(C-5)* A tangent to a circle at one endpoint of a chord is perpendicular to the chord.

always 20. *(B-1)* If two circles have unequal radii, but a central angle of one is congruent to a central angle of the other, then the degree measures of their intercepted arcs are equal.

sometimes 21. *(B-1)* If the degree measure of an arc of one circle is equal to the degree measure of an are of another circle, then the arcs are congruent.

always 22. *(C-7)* If a chord of one circle is congruent to a chord of a congruent circle, then the chords are equidistant from the center of their respective circles.

sometimes 23. *(D-1)* Cyclic triangles are congruent.

always 24. *(D-2)* Regular polygons, triangles, and rectangles are cyclic.

always 25. *(B-1, B-2)* If an inscribed angle and a central angle of a circle intercept the same arc, the measure of the central angle is greater than the measure of the inscribed angle.

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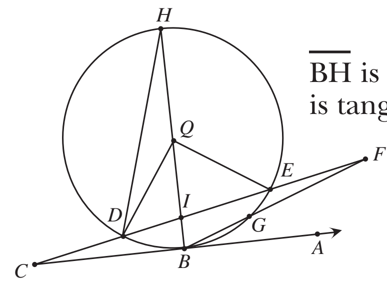

$\overline{BH}$ is a diameter of ⊙Q and $\overline{CA}$ is tangent to ⊙Q at point B.

$m\widehat{EC} = 24$
$m\angle HBG = 76$
$m\angle BQD = 40$

Use the figure to the right and the given information to answer problems 26 to 35.

(B-3)
26. Find m∠ABH m∠ABH = *90*

$m\angle ABH = 90$ (Corollary 68a)

(B-2)
27. Find m∠ABF . m∠ABF = *14*

$$m\angle HBG = \frac{1}{2}\cdot m\widehat{HG} \text{ (Theorem 67)}$$
$$76 = \frac{1}{2}\cdot m\widehat{HG}$$
$$152 = m\widehat{HG}$$
$$m\widehat{BG} = 180 - m\widehat{HG} = 180 - 152 = 28$$

$$m\angle ABF = \frac{1}{2}\cdot m\widehat{BG} \text{ (Theorem 68)}$$
$$= \frac{1}{2}\cdot 28 = \frac{1\cdot \cancel{2}\cdot 14}{\cancel{2}} = 14$$

(B-5)
28. Find m∠ACF m∠ACF = *6*

$$m\widehat{BE} = m\widehat{BG} + m\widehat{EG} = 28 + 24 = 52$$
$$m\widehat{BD} = m\angle BQD = 40$$

$$m\angle ACF = \frac{1}{2}\cdot\left(m\widehat{BE} - m\widehat{BD}\right) \text{ (Theorem 71)}$$
$$= \frac{1}{2}\cdot(52-40) = \frac{1}{2}\cdot 12 = \frac{1\cdot \cancel{2}\cdot 6}{\cancel{2}} = 6$$

(A-2)
29. Find m∠DQH m∠DQH = *140*

$$m\angle DQH = m\widehat{DH}$$
$$m\widehat{DH} = 180 - m\widehat{BD} = 180 - 40 = 140$$
$$m\angle DQH = 140$$

The measure of a central angle is equal to the measure of its intercepted arc.

(A-2)
30. Find m∠BQE m∠BQE = *52*

$$m\angle BQE = m\widehat{BE}$$
$$m\widehat{BE} = m\widehat{EG} + m\widehat{BG} = 24 + 28 = 52$$

The measure of a central angle is equal to the measure of its intercepted arc.

(A-4)
31. Find m∠CFB m∠CFB = *8*

$$m\angle CFB = \frac{1}{2}\left(m\widehat{BD} - m\widehat{EG}\right) \text{ (Theorem 70)}$$
$$= \frac{1}{2}(40-24) = \frac{1}{2}\cdot 16 = \frac{1\cdot \cancel{2}\cdot 8}{\cancel{2}} = 8$$

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(B-4)
32. Find m∠HIE m∠HIE = *84*

$m\widehat{EH} = m\widehat{HG} - m\widehat{EG}$
$= 152 - 24$
$= 128$

$m\angle HIE = \frac{1}{2}\left(m\widehat{BD} + m\widehat{EH}\right)$ (Theorem 69)
$= \frac{1}{2}(40 + 128)$
$= \frac{1}{2} \cdot 168$
$= \frac{1 \cdot \cancel{2} \cdot 84}{\cancel{2}}$
$= 84$

(B-2)
33. Find m∠BDH m∠BDH = *90*

$m\angle BDH = 90$ (Corollary 67a)

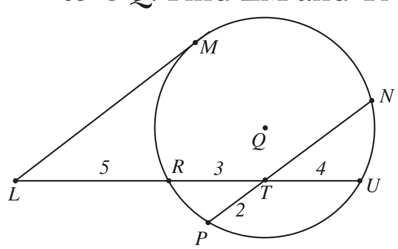
(B-2)
34. Find m∠BHD m∠BHD = *20*

$m\angle BHD = \frac{1}{2} \cdot m\widehat{BD}$ (Theorem 67)
$= \frac{1}{2} \cdot 40$
$= \frac{1 \cdot \cancel{2} \cdot 20}{\cancel{2}}$
$= 20$

(B-2)
35. Find m∠HDQ m∠HDQ = *20*

$mHDQ = m\angle DHQ$
$\triangle QHD$ is Isosceles.
$m\angle BHD = m\angle DHQ$ (refer to exercise 34)
$m\angle HDQ = 20$

(C-4)
36. In the figure below $\overline{LM}$ is tangent to ⊙Q. Find LM and TN.

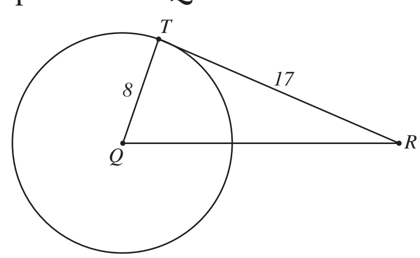


LM = $2\sqrt{15}$
TN = *6*

$\frac{UL}{LM} = \frac{LM}{LR}$ (Theorem 78)
$\frac{(5+3+4)}{LM} = \frac{LM}{5}$
$\frac{12}{LM} = \frac{LM}{5}$
$(LM)(LM) = 12 \cdot 5$
$LM^2 = 60$
$LM = \pm\sqrt{60}$ (LM cannot be negative)
$LM = \sqrt{4} \cdot \sqrt{15}$
$LM = 2\sqrt{15}$

$TN \cdot TP = RT \cdot TU$
$TN \cdot 2 = 3 \cdot 4$
$TN \cdot 2 = 12$
$TN = 6$

(B-3)
37. In the figure below, $\overleftrightarrow{RT}$ is tangent to ⊙Q at point T. If QT = 8 and RT = 17. Find RQ.



RQ = $\sqrt{353}$

∠RTQ is a right angle so △RTQ is a right triangle (Corollary 68a)

$(RT)^2 + (QT)^2 = (RQ)^2$
$(17)^2 + (8)^2 = (RQ)^2$
$289 + 64 = (RQ)^2$
$353 = (RQ)^2$
$\pm\sqrt{353} = RQ$ (RQ cannot be negative)
$\sqrt{353} = RQ$

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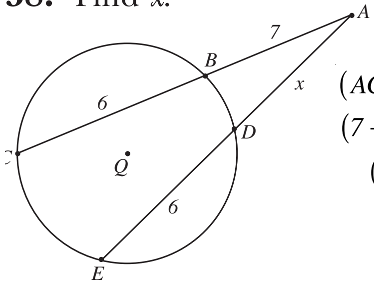
(C-4)
38. Find x. $x =$ <u>7</u>



$(AC)(AB) = (AE)(AD)$ *(Theorem 77)*

$(7+6)(7) = (x+6)(x)$

$(13)(7) = x^2 + 6x$

$91 = x^2 + 6x$

$0 = x^2 + 6x - 91$

$0 = (x+13)(x-7)$

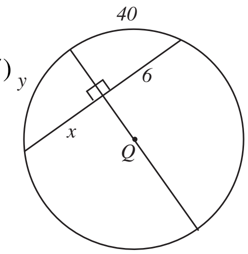
$0 = x + 13 \text{ or } 0 = x - 7$

$-13 = x \text{ or } 7 = x$

(x cannot be negative)

$7 = x$

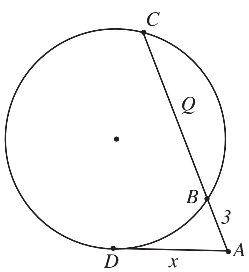
(C-1)
39. Find x and y $x =$ <u>6</u> $y =$ <u>40</u>



$x = 6$ *(Theorem 73)*

$y = 40$ *(Corollary 73a)*

(C-4)
40. Find x. $x =$ <u>$2\sqrt{6}$</u>



$\frac{AC}{AD} = \frac{AD}{AB}$ *(Theorem 78)*

$\frac{(5+3)}{x} = \frac{x}{3}$

$\frac{8}{x} = \frac{x}{3}$

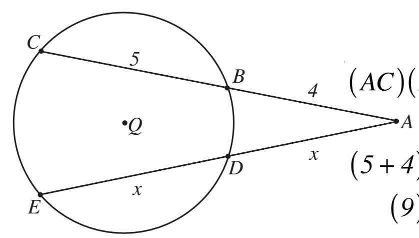
$x^2 = 24$

$x = \pm\sqrt{24}$ *(x cannot be negative)*

$x = \sqrt{4} \cdot \sqrt{6}$

$x = 2\sqrt{6}$

(C-4)
41. Find x. $x =$ <u>$3\sqrt{2}$</u>



$(AC)(AB) = (AE)(AD)$

(Theorem 77)

$(5+4)(4) = (x+x)(x)$

$(9)(4) = (2x)(x)$

$36 = 2x^2$

$18 = x^2$

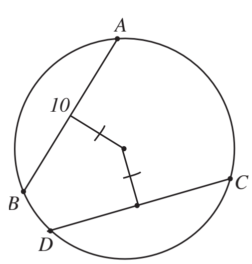
$\pm\sqrt{18} = x$

(x cannot be negative)

$\sqrt{9} \cdot \sqrt{2} = x$

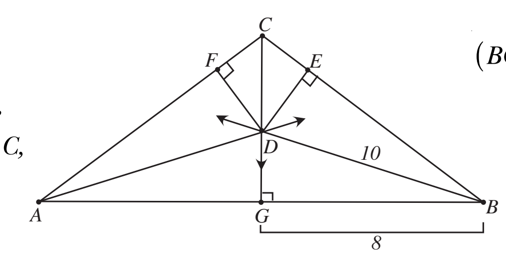
$3\sqrt{2} = x$

(C-7)
42. Find DC DC = <u>10</u>



DC = 10
Two chords of the same circle equidistant from the center are congruent. (See Unit VI, Part C, Lesson 7, Exercise 3)

(D-1)
43. Find DF. DF = <u>6</u>



$(BG)^2 + (DG)^2 = (BD)^2$

$8^2 + (DG)^2 = 10^2$

$64 + (DG)^2 = 100$

$(DG)^2 = 36$

$DG = \pm\sqrt{36}$

(DG cannot be negative)

$DG = 6$

$DF = 6$

Angle bisectors meet at a point equidistant from the sides of the triangle (Corollary 83b)

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