Geometry: A Complete Course
(with Trigonometry)

Module E - Course Notes

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VideoText Interactive

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Module E - Course Notes

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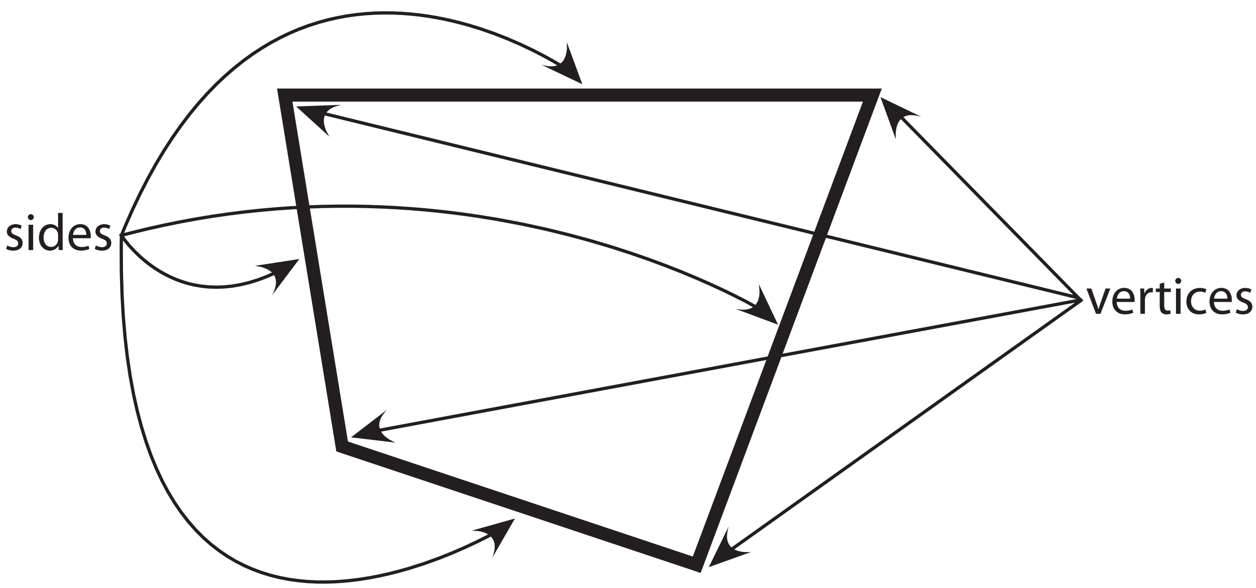
Course Notes

QUADRILATERAL

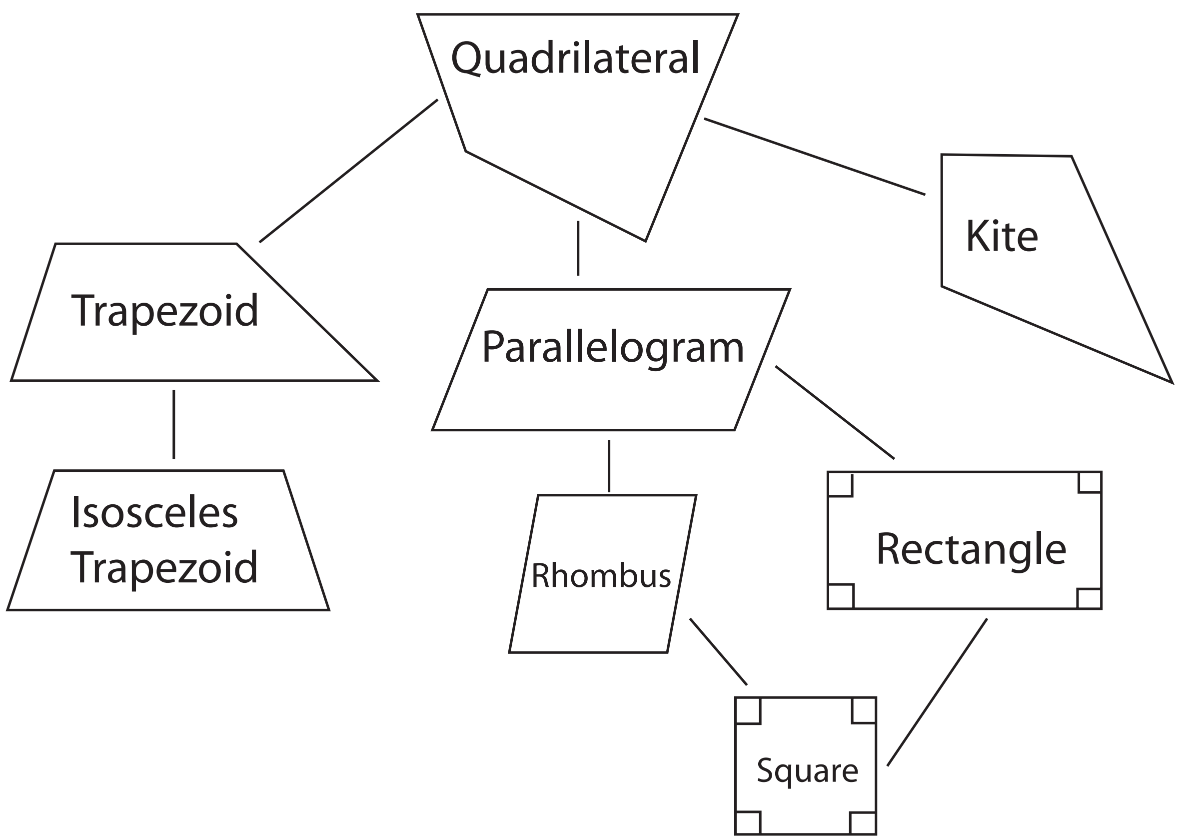
"A simple closed plane curve, made up of four straight line segments."



COURSE NOTE 237

QUADRILATERAL HIERARCHY THEOREM

"If a polygon is one of the seven types of quadrilaterals, then it is related to all other quadrilaterals using the diagram below."



COURSE NOTE 238

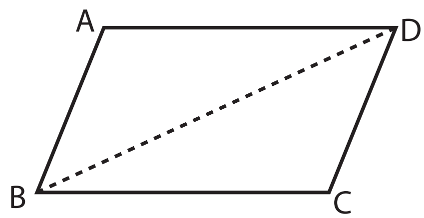
THEOREM 41

1) "If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent."

3) Given: ▱ABCD

4) Prove: $\overline{DC} \cong \overline{BA}$; $\overline{AD} \cong \overline{CB}$

2)



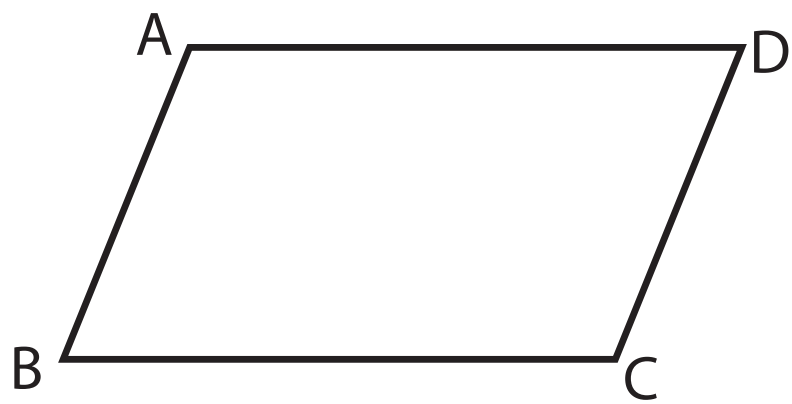
5) Analysis: Auxiliary Line, Definition of Congruent Triangles

6) STATEMENT	REASON
1. ▱ABCD	1. Given
2. Draw diagonal $\overline{DB}$	2. Postulate 2
3. $\overline{DC} \parallel \overline{BA}$	3. Definition of Parallelogram
4. $\overline{AD} \parallel \overline{CB}$	4. Definition of Parallelogram
5. $\angle 1 \cong \angle 4$	5. Theorem 16
6. $\angle 2 \cong \angle 3$	6. Theorem 16
7. $\overline{DB} \cong \overline{DB}$	7. Reflexivity of Congruence
8. $\triangle ABD \cong \triangle CDB$	8. A.S.A. Congruence Assumption
9. $\overline{DC} \cong \overline{BA}$	9. C.P.C.T.C.
10. $\overline{AD} \cong \overline{CB}$	10. C.P.C.T.C. (Q.E.D.)

COURSE NOTE 239

THEOREM 42

"If a quadrilateral is a parallelogram, then any pair of consecutive angles are supplementary"

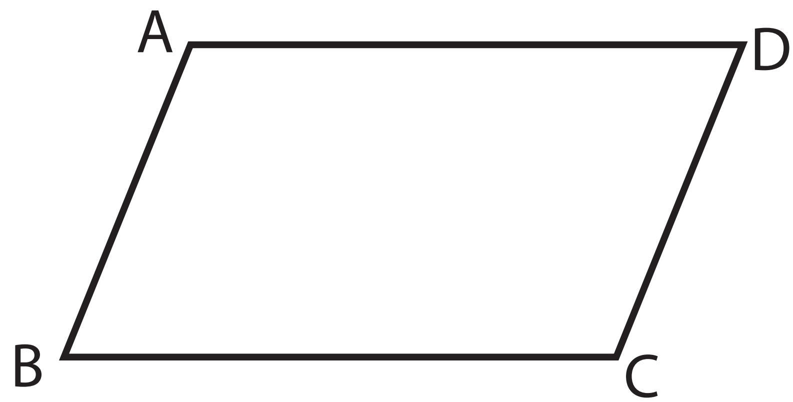


For parallelogram ADCB,

$\angle$A is supplementary to $\angle$D
$\angle$D is supplementary to $\angle$C
$\angle$C is supplementary to $\angle$B
$\angle$B is supplementary to $\angle$A

COROLLARY 42a

"If a quadrilateral is a parallelogram, then opposite angles are congruent."



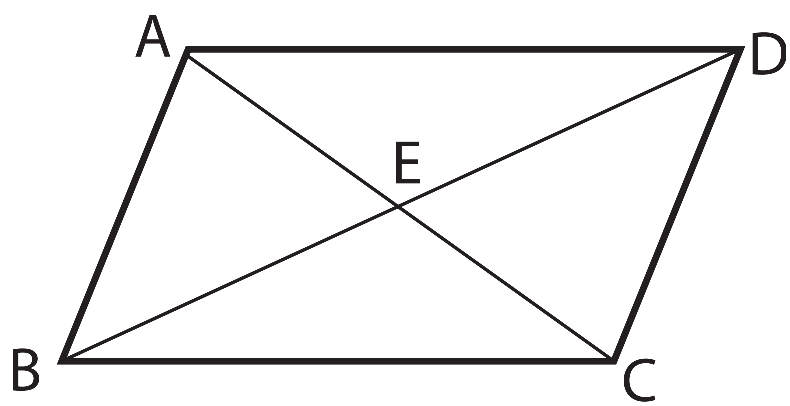
For parallelogram ADCB,

$\angle A \cong \angle C$

$\angle B \cong \angle D$

THEOREM 43

"If a quadrilateral is a parallelogram, then its diagonals bisect each other."



For parallelogram ADCB,with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E,

$$\overline{AE} \cong \overline{CE}$$
$$\overline{BE} \cong \overline{DE}$$

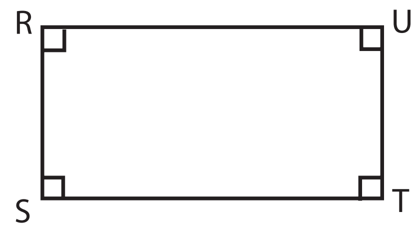
THEOREM 48

1) "If a quadrilateral is a rectangle, then it is a parallelogram."

3) Given: ▭ RSTU

4) Prove: ▭ RSTU is a parallelogram

2)



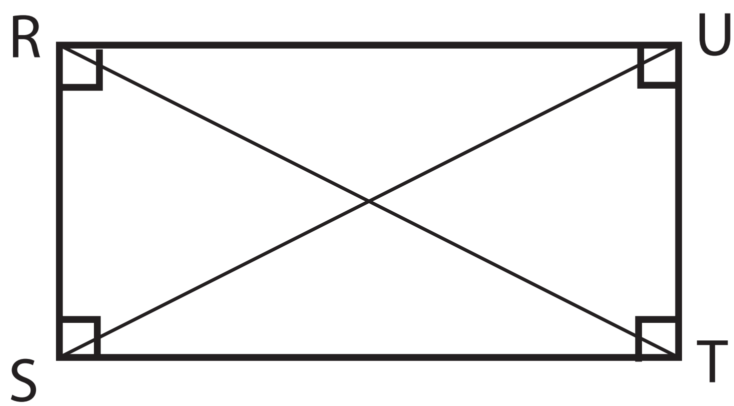
5) Analysis: Theorem 46

6) STATEMENT	REASON
1. ▭ RSTU	1. Given
2. ∠R, ∠S, ∠T, and ∠U are right angles	2. Definition of a Rectangle
3. $\angle R \cong \angle T$	3. Theorem 11
4. $\angle S \cong \angle U$	4. Theorem 11
5. ▭ RSTU is a parallelogram	5. Theorem 46

COURSE NOTE 243

THEOREM 49

"If a quadrilateral is a rectangle, then its diagonals are congruent."



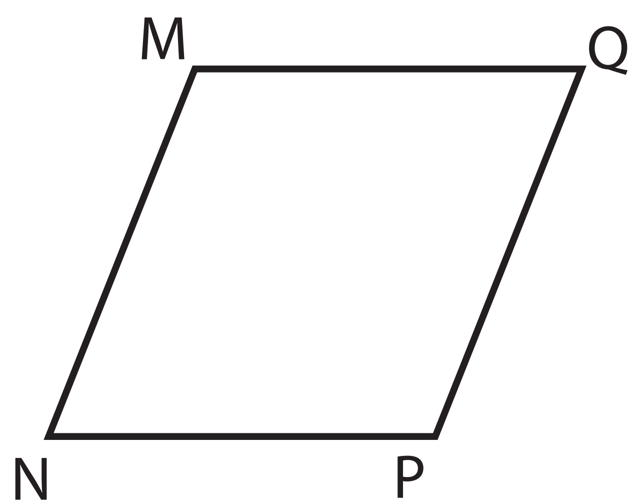
For rectangle RUTS, with diagonals $\overline{RT}$ and $\overline{SU}$,

$$\overline{RT} \cong \overline{SU}$$

COURSE NOTE 244

THEOREM 50

"If a quadrilateral is a rhombus, then it is a rectangle."

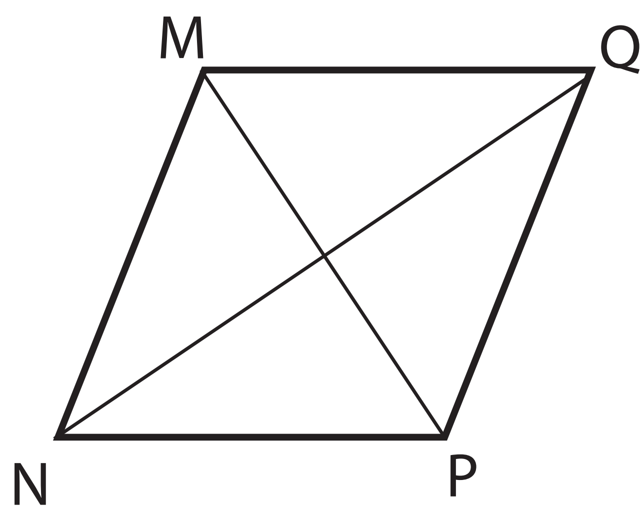


For rhombus MQPN,
MQPN is a parallelogram

COURSE NOTE 245

THEOREM 51

"If a quadrilateral is a rhombus, then its diagonals are perpendicular."



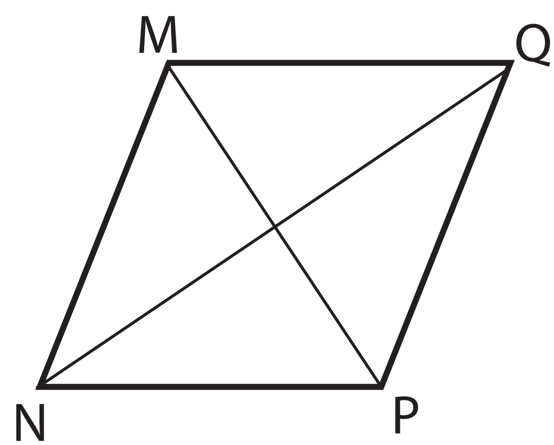
For rhombus MQPN,
with diagonals $\overline{MP}$ and $\overline{NQ}$,

$$\overline{MP} \perp \overline{NQ}$$

COURSE NOTE 246

THEOREM 52

"If a quadrilateral is a rhombus, then each diagonal bisects a pair of opposite angles of the rhombus."



For rhombus MQPN
with diagonals $\overline{MP}$ and $\overline{NQ}$,

$\angle MQN \cong \angle PQN$

$\angle MNQ \cong \angle PNQ$

$\angle NMP \cong \angle QMP$

$\angle NPM \cong \angle QPM$

COURSE NOTE 247

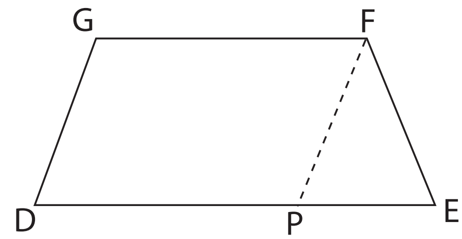
THEOREM 53

1) "If you have an isosceles trapezoid, then its base angles are congruent."

3) Given: Isosceles Trapezoid DEFG, with bases $\overline{DE}$ and $\overline{FG}$

4) Prove: $\angle D \cong \angle E$; $\angle F \cong \angle G$

2)



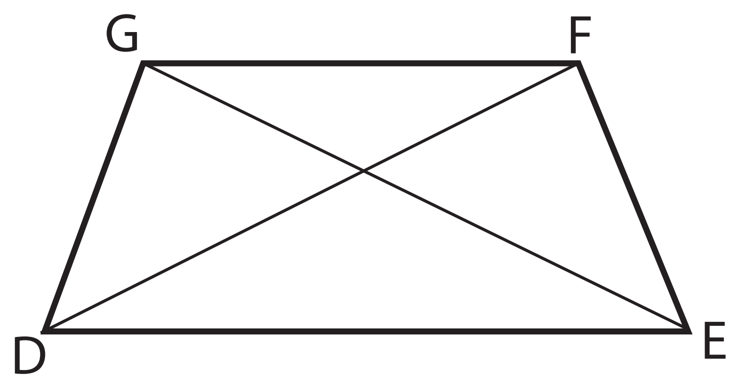
5) Analysis: Postulate 9, Definition of a Parallelogram

6) STATEMENT	**REASON**
1. Isosceles Trapezoid DEFG, with bases $\overline{DE}$ and $\overline{FG}$	1. Given
2. $\overline{DE}$ // $\overline{FG}$	2. Definition of a Trapezoid
3. Draw $\overline{FP}$ // $\overline{DG}$	3. Postulate 9
4. Quadrilateral DPFG is a parallelogram	4. Theorem 11
5. $\overline{DG} \cong \overline{PF}$	5. Theorem 41
6. $\overline{DG} \cong \overline{EF}$	6. Definition of Isosceles Trapezoid
7. $\overline{PF} \cong \overline{EF}$	7. Substitution
8. $\angle FPE \cong \angle E$	8. Theorem 33
9. $\angle FPE \cong \angle D$	9. Postulate 11
10. $\angle D \cong \angle E$	10. Substitution
11. $\angle D$ is supplementary to $\angle G$	11. Theorem 17
12. $\angle E$ is supplementary to $\angle F$	12. Theorem 17
13. $\angle F \cong \angle G$	13. Theorem 14 (Q.E.D.)

COURSE NOTE 248

COROLLARY 53a

"If you have an isosceles trapezoid, then its diagonals are congruent."



For isosceles trapezoid GFED with diagonals $\overline{GE}$ and $\overline{DF}$,

$$\overline{GE} \cong \overline{DF}$$

COURSE NOTE 249

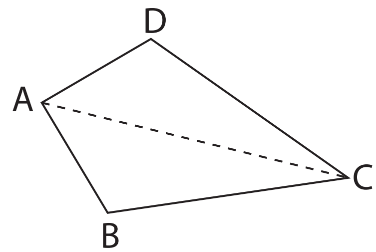
THEOREM 54

1) "If a quadrilateral is a kite, then the pair of opposite angles formed by the pair of non-congruent sides, are congruent."

3) Given: Kite ABCD, with $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$

4) Prove: $\angle B \cong \angle D$

2)



5) Analysis: Auxiliary Line, Definition of Congruent Triangles (C.P.C.T.C.)

6) STATEMENT	REASON
1. Kite ABCD, with $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$	1. Given
2. Draw diagonal $\overline{AC}$	2. Postulate 2
3. $\overline{AC} \cong \overline{AC}$	3. Reflexivity of Congruence
4. $\triangle ABC \cong \triangle ADC$	4. S.S.S. Congruence Assumption
5. $\angle B \cong \angle D$	5. C.P.C.T.C. (Q.E.D.)

COURSE NOTE 250

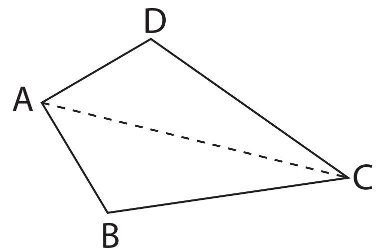
COROLLARY 54a

1) "If a quadrilateral is a kite, then the longest diagonal bisects the angles to which it is drawn"

3) Given: Kite ABCD, with $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$

4) Prove: $\overline{AC}$ bisects $\angle A$ and $\angle C$

2)



5) Analysis: Auxiliary Line, Definition of Congruent Triangles (C.P.C.T.C.)

6) STATEMENT	**REASON**
⋮	⋮
(continued from Theorem 54)	(continued from Theorem 54)
⋮	⋮
6. $\angle 1 \cong \angle 2$	6. C.P.C.T.C.
7. $\angle 3 \cong \angle 4$	7. C.P.C.T.C.
8. $\overline{AC}$ bisects $\angle A$ and $\angle C$	8. Definition of Angle Bisector (Q.E.D.)

COURSE NOTE 251

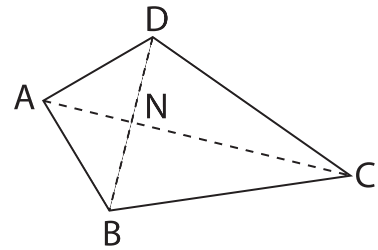
COROLLARY 54b

1) "If a quadrilateral is a kite, then the longer diagonal bisects the shorter diagonal."

3) Given: Kite ABCD, with $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$

4) Prove: $\overline{AC}$ bisects $\overline{BD}$

2)



5) Analysis: Auxiliary Line, Definition of Congruent Triangles (C.P.C.T.C.)

6) STATEMENT	**REASON**
⋮	⋮
(continued from Corollary 54a)	(continued from Corollary 54a)
⋮	⋮
9. Draw $\overline{BD}$, intersecting $\overline{AC}$ at N	9. Postulate 2
10. $\overline{AN} \cong \overline{AN}$	10. Reflexivity of Congruence
11. $\triangle ABN \cong \triangle AND$	11. S.A.S. Congruence Assumption
12. $\overline{BN} \cong \overline{DN}$	12. C.P.C.T.C.
13. N is the midpoint of $\overline{BD}$	13. Definition of Midpoint
14. $\overline{AC}$ bisects $\overline{BD}$	14. Definition of Segment Bisector (Q.E.D.)

COURSE NOTE 252

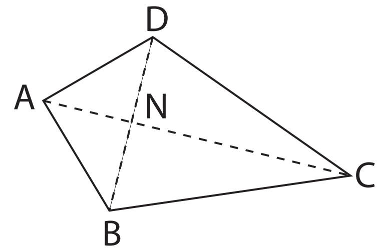
COROLLARY 54c

1) "If a quadrilateral is a kite, then the diagonals are perpendicular to each other."

3) Given: Kite ABCD, with $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$

4) Prove: $\overline{AC} \perp \overline{BD}$

2)



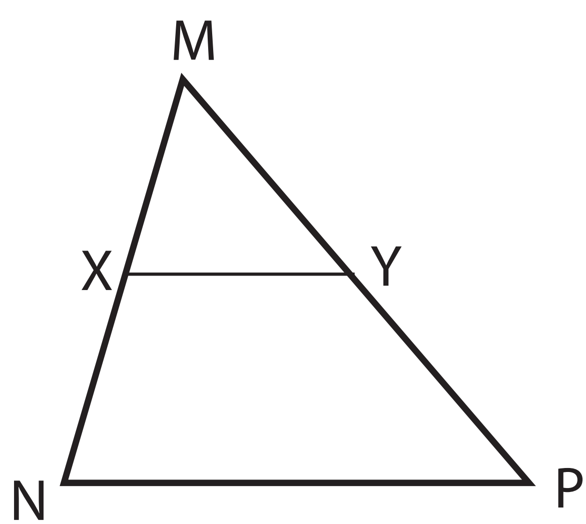
5) Analysis: Auxiliary Line, Definition of Congruent Triangles (C.P.C.T.C.)

6) STATEMENT	REASON
• • •	• • •
(continued from Corollary 54b)	(continued from Corollary 54b)
• • •	• • •
15. $\angle ANB \cong \angle AND$	15. C.P.C.T.C.
16. $\angle ANB$ and $\angle AND$ are a linear pair	16. Definition of Linear Pair
17. $\angle BND$ is a straight angle	17. Definition of Straight Angle
18. $m\angle BND = 180$	18. Definition of Straight Angle
19. $m\angle ANB + m\angle AND = m\angle BND$	19. Postulate 7 (Protractor) - Angle Addition Assumption
20. $m\angle ANB + m\angle AND = 180$	20. Substitution
21. $m\angle ANB = m\angle AND$	21. Definition of Congruent Angles
22. $m\angle ANB + m\angle ANB = 180$	22. Substitution
23. $2m\angle ANB = 180$	23. Arithmetic Fact
24. $m\angle ANB = 90$	24. Multiplication of Equality
25. $\angle ANB$ is a right angle	25. Definition of Right Angle
26. $\overline{AC} \perp \overline{BD}$	26. Definition of Perpendicular Lines (Q.E.D.)

COURSE NOTE 253

MIDSEGMENT

"A line segment which joins the midpoints of two sides of any polygon."



$\overline{XY}$ is a midsegment of $\triangle MNP$, joining sides $\overline{MN}$ and $\overline{MP}$

COURSE NOTE 254

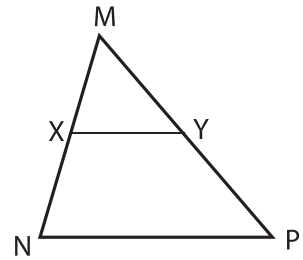
THEOREM 55

1) "If you have two triangles, then the midsegment joining two sides, is parallel to the third side, and is one-half the measure of that side."

3) Given: △MNP, with midsegment XY joining MN and MP

4) Prove: $\overline{XY} \parallel \overline{NP}$, and XY = ½ NP

2)



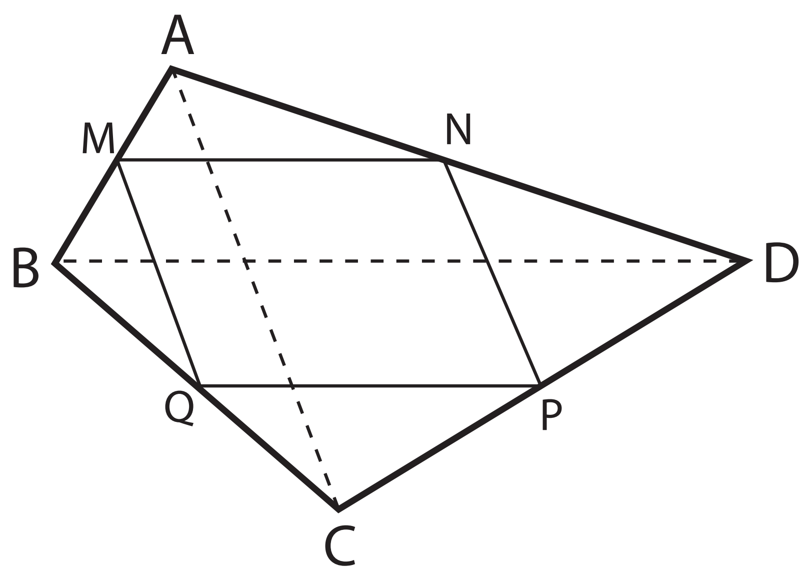
5) Analysis: Similar Triangles, Theorem 19

6) **STATEMENT**	**REASON**
1. △MNP, with midsegment XY joining MN and MP	1. Given
2. X is the midpoint of MN	2. Definition of Midsegment
3. MX ≅ XN	3. Definition of Midpoint
4. MX = XN	4. Definition of Congruent Segments
5. MX + XN = MN	5. Postulate 6(Ruler)-Segment-Addition
6. MX + MX = MN	6. Substitution
7. 2MX = MN	7. Arithmetic Fact
8. MX = ½MN	8. Multiplication of Equality
9. $\frac{MX}{MN} = \frac{1}{2}$	9. Multiplication of Equality
10. Y is the midpoint of MP	10. Definition of Midsegment
11. MY ≅ YP	11. Definition of Midpoint
12. MP = YP	12. Definition of Congruent Segments
13. MY + YP = MP	13. Postulate 6(Ruler)-Segment-Addition
14. MY + MY = MP	14. Substitution
15. 2MY = MP	15. Arithmetic Fact
16. MY = ½MP	16. Multiplication of Equality
17. $\frac{MX}{MN} = \frac{1}{2}$	17. Multiplication of Equality
18. ∠M ≅ ∠M	18. Reflexive Property of Congruence
19. △MXY ~ △MNP	19. S.A.S. Similiarity Assumption
20. ∠MXY ≅ ∠N	20. Definition of Similarity
21. XY ∥ NP	21. Theorem 19
22. $\frac{MX}{MN} = \frac{1}{2}$	22. Definition of Similarity
23. XY = ½ NP	23. Multiplication of Equality (Q.E.D.)

COURSE NOTE 255

THEOREM 56

"If you have a quadrilateral, then the four midsegments joining consecutive sides of that quadrilateral, form a parallelogram."



$\overline{MN}$, $\overline{NP}$, $\overline{PQ}$ and $\overline{QM}$ are midsegments of quadrilateral ABCD.

Quadrilateral MNPQ is a parallelogram.

POLYGONAL REGION

"The set of points inside a polygon"

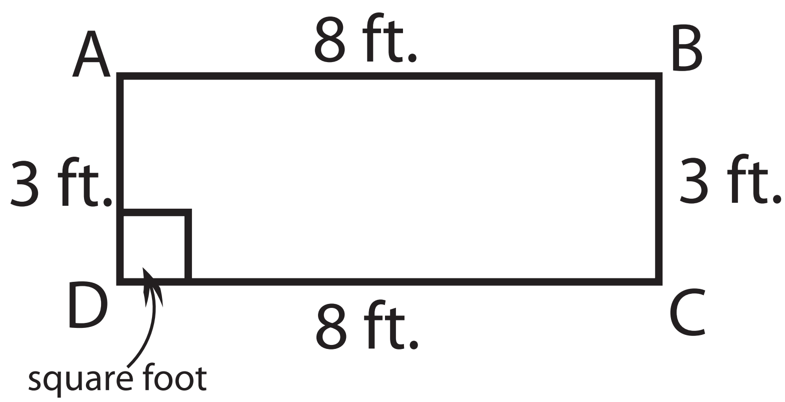
"If you have a polygonal region, then you have the set of points inside a polygon"

"If you have the set of points inside a polygon, then you have a polygonal region."

COURSE NOTE 257

POSTULATE 14: Area (1st Assumption)

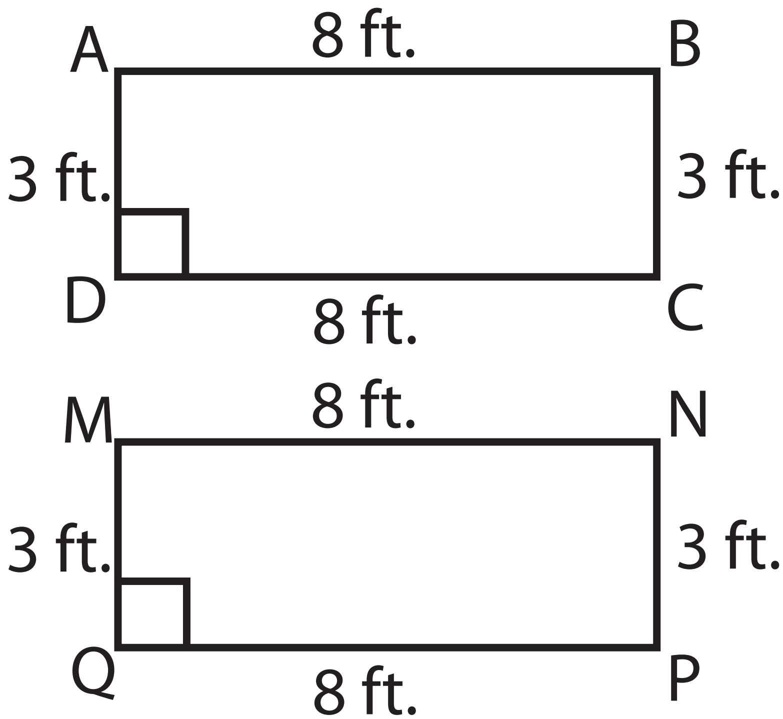
"To every polygonal region, there corresponds a unique positive real number, called its area, relative to a specific unit."



Area of ▭ABCD is 24 square feet (relative to the specified unit)

POSTULATE 14: Area (2nd Assumption)

"Congruent polygons have the same area."



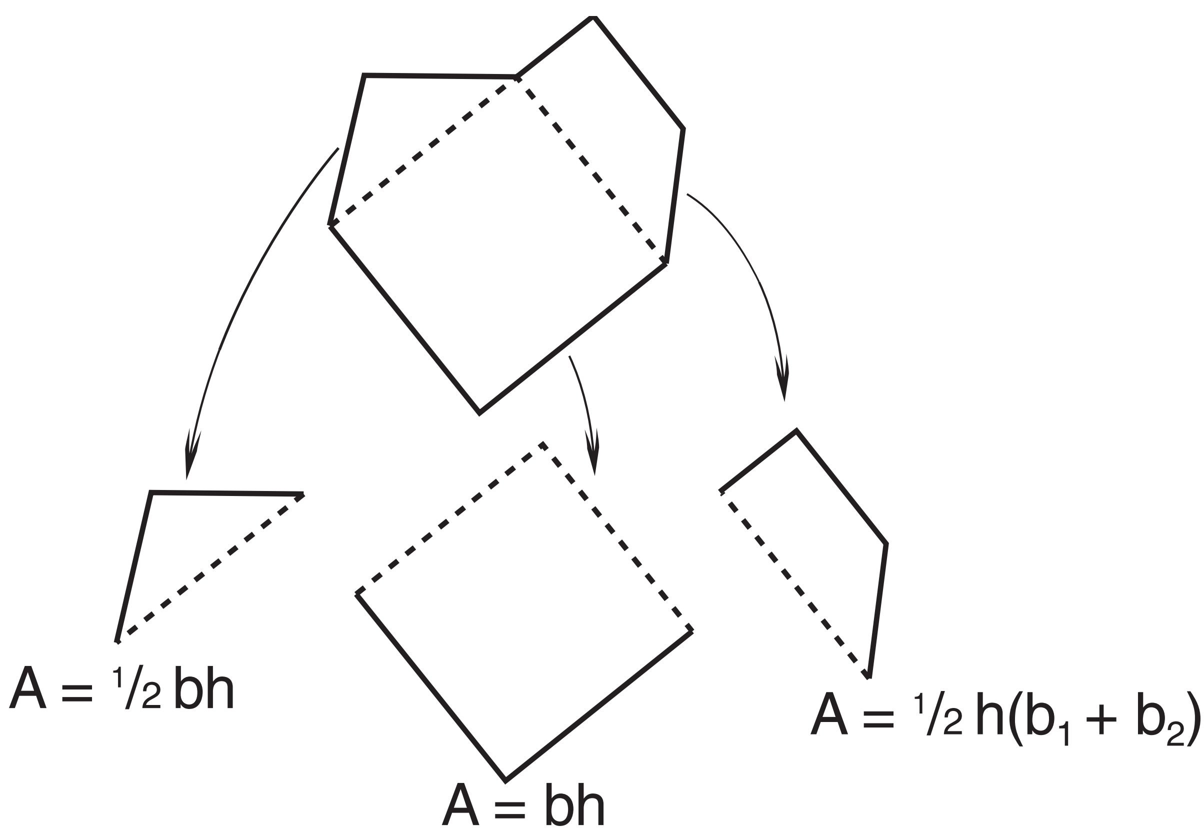
Area of ▭ABCD = Area of ▭MNPQ

COURSE NOTE 259

POSTULATE 14: Area (3rd Assumption)

"If a plane figure can be separated into a number of non-overlapping polygonal regions, then the area of that plane figure, is the sum of the areas of those polygonal regions."

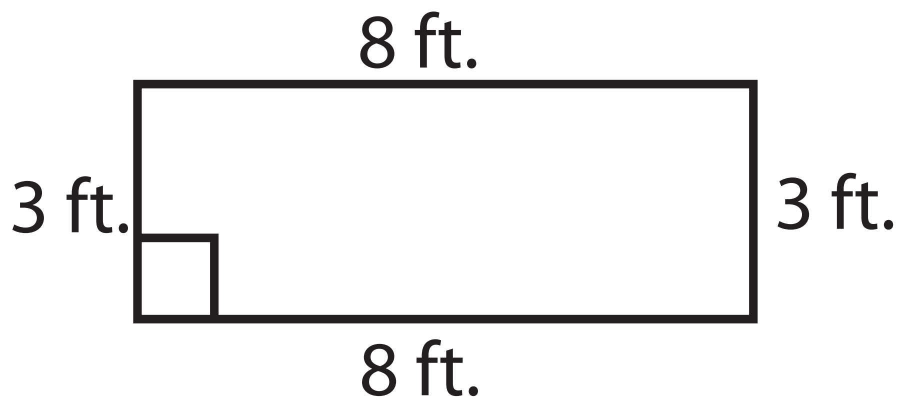
(Area-Addition Assumption)



COURSE NOTE 260

POSTULATE 14: Area (4th Assumption)

"The area *A* of a rectangular region, with dimensions *b* (for the base), and *h* (for the height), is $b \cdot h$ (or base • height)

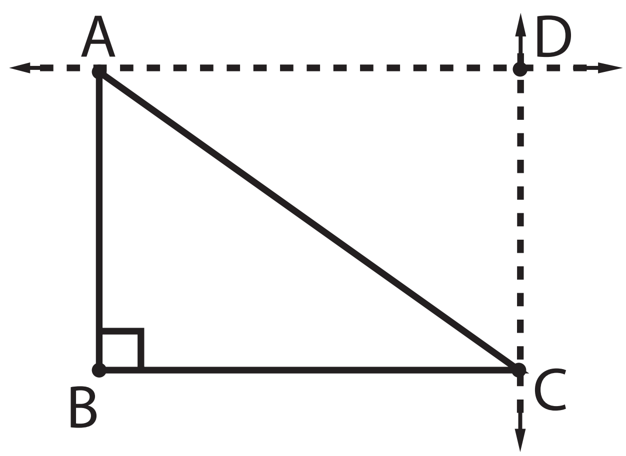


Area = base • height
24 = 8 • 4

The area is 24 square feet.

SUPPORTING ARGUMENT
("make clear" or "prove")
for
THEOREM 58

"If you have a right triangle, then the area inside the triangle, is one-half the product of the legs."



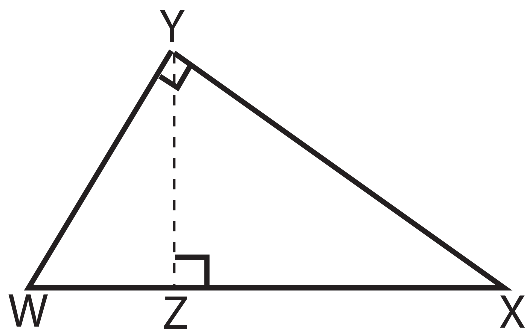
The area inside $\triangle ABC = \frac{1}{2} \cdot AB \cdot BC$

Using: Postulate 9 (parallel lines)
Definition of parallelogram
Theorem 42 (consecutive angles of a ▱)
Corollary 42a (opposite angles of a ▱)
Definition of a rectangle
Postulate 14: Area

COURSE NOTE 262

SUPPORTING ARGUMENT
("make clear" or "prove")
for
COROLLARY 58a

"If you have a triangle, then the area inside the triangle, is one-half the product of the measures of the base and the altitude."



$$\text{Area} = \frac{1}{2}(WZ \cdot YZ) + \frac{1}{2}(XZ \cdot YZ)$$
$$= \frac{1}{2}YZ(WZ + XZ)$$
$$= \frac{1}{2}YZ \cdot WX$$
$$= \frac{1}{2} \cdot b \cdot h$$

Using: Postulate 14: (Area-Addition Assumption)
Theorem 58
Postulate 6: Ruler

COURSE NOTE 263

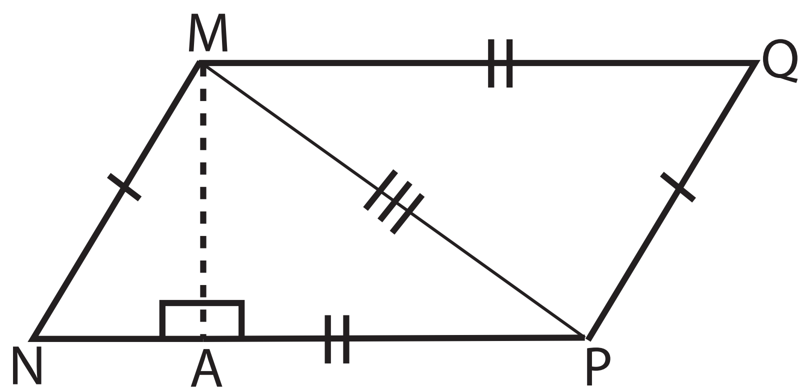
SUPPORTING ARGUMENT

("make clear" or "prove")

for

THEOREM 59

"If you have a parallelogram, then the area inside the parallelogram, is the product of the measures of any base, and the corresponding altitude."



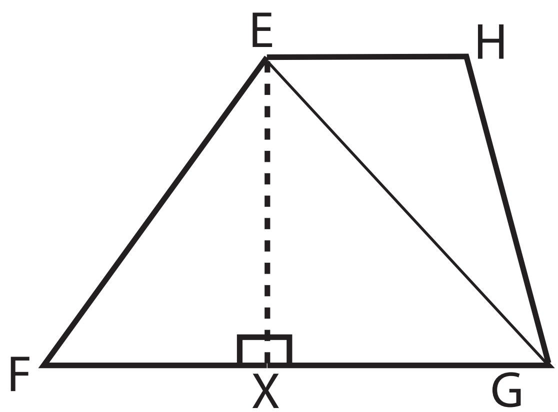
$$\text{Area} = \frac{1}{2} \cdot NP \cdot AM \cdot 2$$
$$= NP \cdot AM$$
$$= b \cdot h$$

Using: Theorem 41
Reflexivity of Congruence
S.S.S. Congruence Assumption
Postulate 14: Area

COURSE NOTE 264

SUPPORTING ARGUMENT
("make clear" or "prove")
for
THEOREM 60

"If you have a trapezoid, then the area inside the trapezoid, is one-half the product of the measure of the altitude, and the sum of the lengths of its bases."



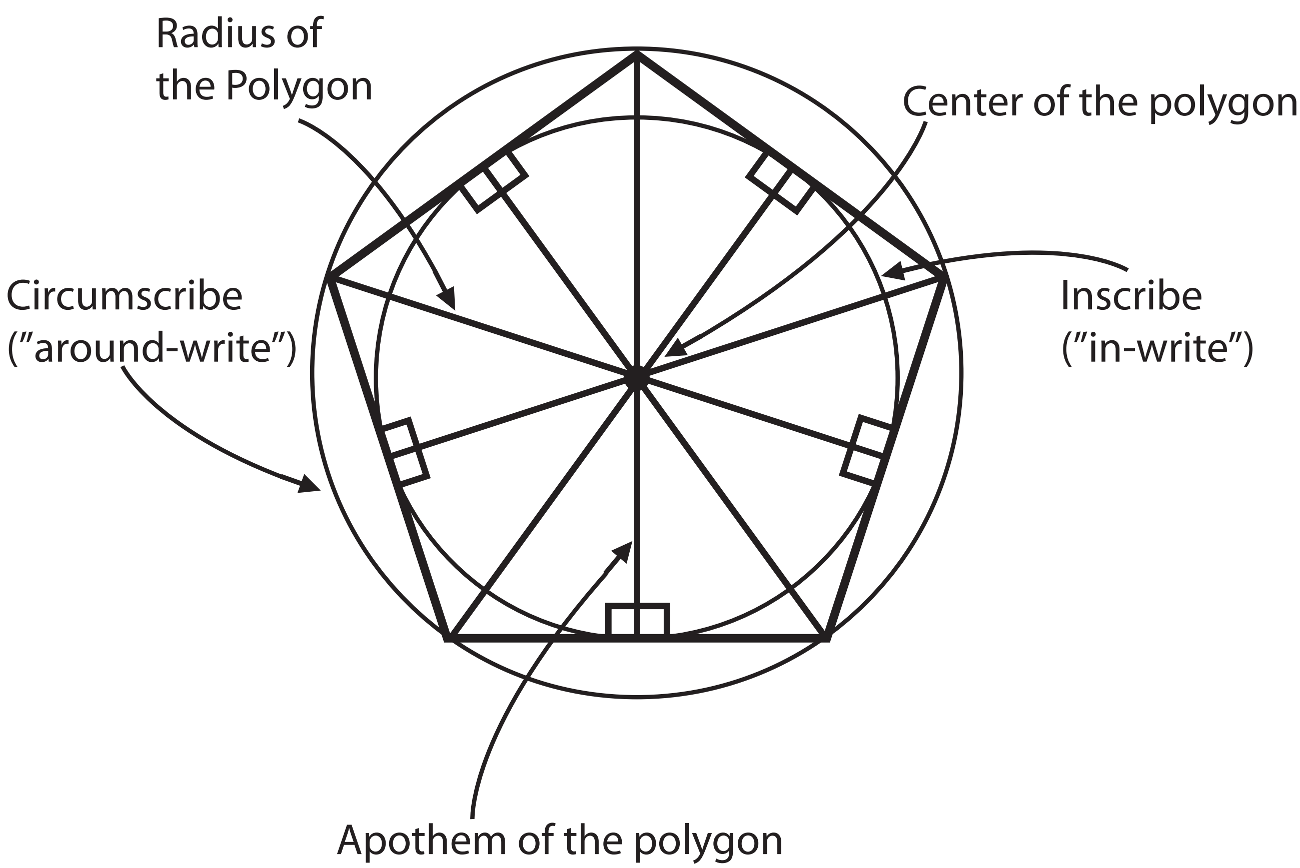
$$\text{Area} = \frac{1}{2}\cdot FG\cdot EX + \frac{1}{2}\cdot EH\cdot EX$$
$$= \frac{1}{2}\cdot EX(FG + EH)$$
$$= \frac{1}{2}\cdot h(b_1 + b_2)$$

Using: Postulate 14: Area
Theorem 58

COURSE NOTE 265

REGULAR POLYGON

"A polygon in which all of the sides are of equal measure, and all of the angles are of equal measure" (equilateral and equiangular)



COURSE NOTE 266

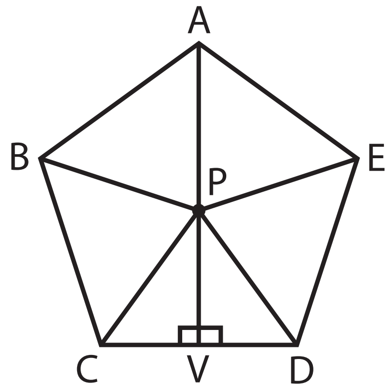
SUPPORTING ARGUMENT

(“make clear” or “prove”)

for

THEOREM 61

“If you have a regular *n*-gon, with sides of length *s*, and apothem of length *a*, then the area *A* inside the *n*-gon, is one-half the product of *s* and *a* and *n*, or one-half the product of *a* and *P*, where *P* is the perimeter of the regular *n*-gon.”



$$\text{Area} = \frac{1}{2}\cdot CD\cdot PV\cdot 5$$

$$= \frac{1}{2}\cdot s\cdot a\cdot n$$

$$= \frac{1}{2}\cdot a\cdot P$$

Using: Corollary 58a
Postulate 14

COURSE NOTE 267

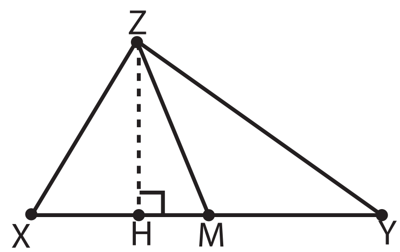
Theorem 62

1) "If you have a median of a triangle, then that median separates the points inside the triangle into two polygonal regions with the same area."

3) Given: $\triangle XYZ$, with median $\overline{ZM}$. drawn to $\overline{XY}$ at midpoint M

4) Prove: Area of $\triangle XMZ$ = Area of $\triangle YMZ$

2)

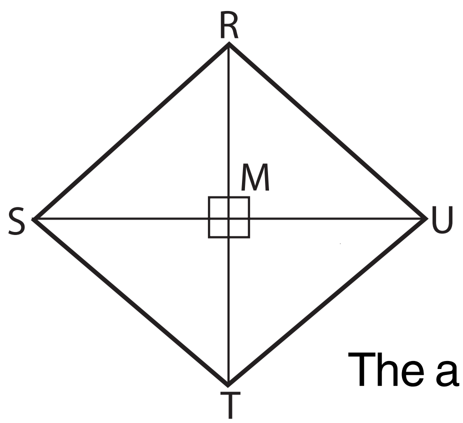


5) Analysis: Corollary 58a

6) STATEMENT	**REASON**
1. $\triangle XYZ$, with median $\overline{ZM}$	1. Given
2. Draw $\overline{ZH} \perp \overline{XY}$	2. Postulate 10
3. $\overline{ZH}$ is the altitude of $\triangle XYZ$	3. Definition of Altitude
4. $\overline{ZH}$ is the altitude of $\triangle XMZ$	4. Definition of Altitude
5. $\overline{ZH}$ is the altitude of $\triangle YMZ$	5. Definition of Altitude
6. Area of $\triangle$ XMZ = $\frac{1}{2} \bullet XM \bullet ZH$	6. Corollary 58a
7. Area of $\triangle$ YMZ = $\frac{1}{2} \bullet YM \bullet ZH$	7. Corollary 58a
8. $\overline{XM} \cong \overline{YM}$	8. Definition of Midpoint
9. XM = YM	9. Definition of Congruent Segments
10. Area of $\triangle$ YMZ = $\frac{1}{2} \bullet XM \bullet ZH$	10. Substitution
11. Area of $\triangle$ XMZ = Area of $\triangle$ YMZ	11. Substitution (Q.E.D.)

SUPPORTING ARGUMENT
("make clear" or "prove")
for
THEOREM 63

"If you have a rhombus, then the area inside the rhombus, is equal to one-half the product of the measures of the diagonals of the rhombus."



$$\triangle RMS \cong \triangle SMT \cong TMU \cong UMR$$

$$\text{The area of } \triangle RMS = \frac{1}{2} \cdot SM \cdot RM$$

$$\text{The area of } RSTU = 4 \cdot \frac{1}{2} \cdot SM \cdot RM$$

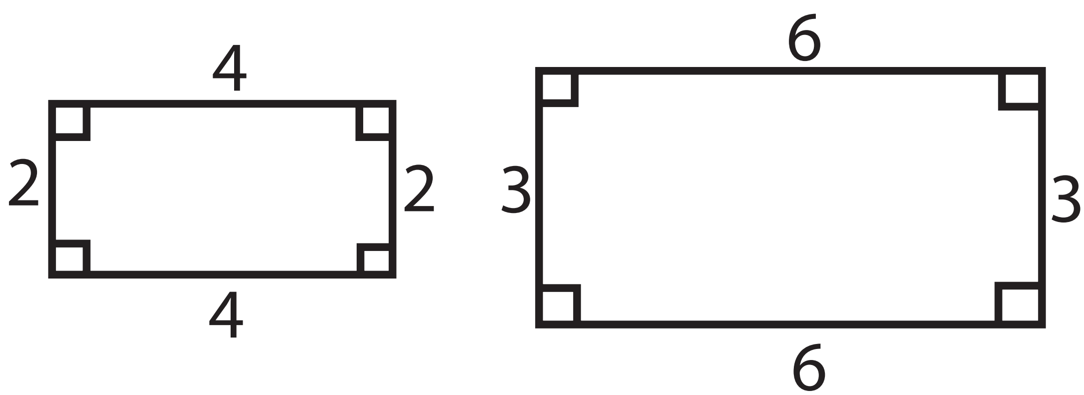
$$= \cancel{4} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{2} \cdot SU \cdot \frac{1}{\cancel{2}} \cdot RT$$

$$= \frac{1}{2} \cdot SU \cdot RT$$

Using: Theorem 51
Theorem 43
L.L. Congruence Postulate Corollary
Corollary 58a

THEOREM 64

"If you have two similar polygons, then the ratio of the areas of the two polygons, is equal to the square of the ratio of any pair of corresponding sides."

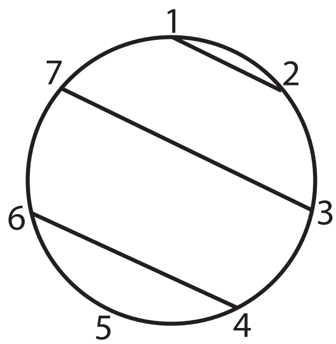


Area = 2 • 4
= 8 sq. units

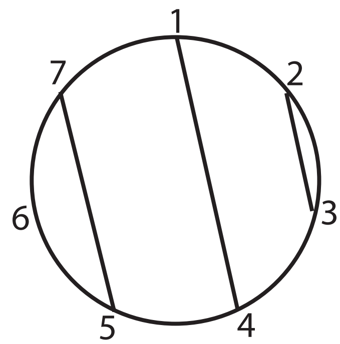
Area = 3 • 6
= 18 sq. units

$$\frac{8}{18} = \frac{4}{9} = \frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2$$

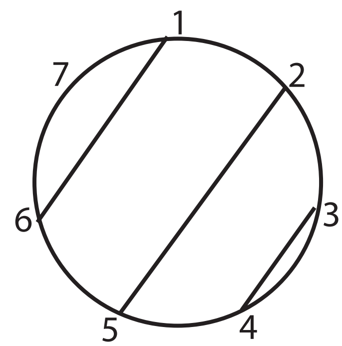
ROUND-ROBIN PAIRINGS
(for an odd number of contestants)



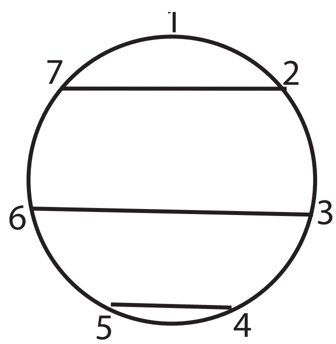
Week 1



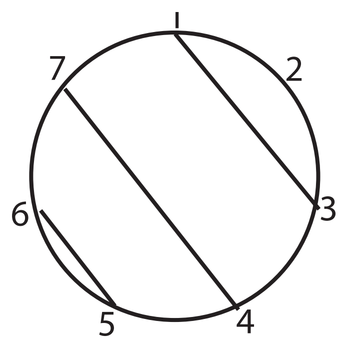
Week 2



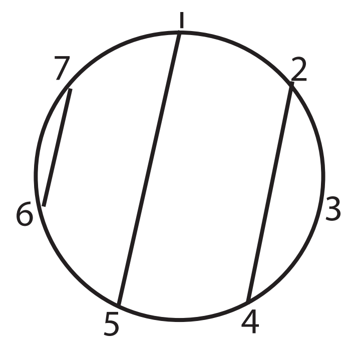
Week 3



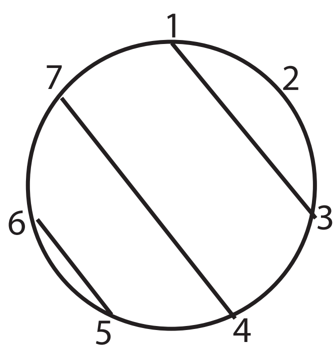
Week 4



Week 5



Week 6

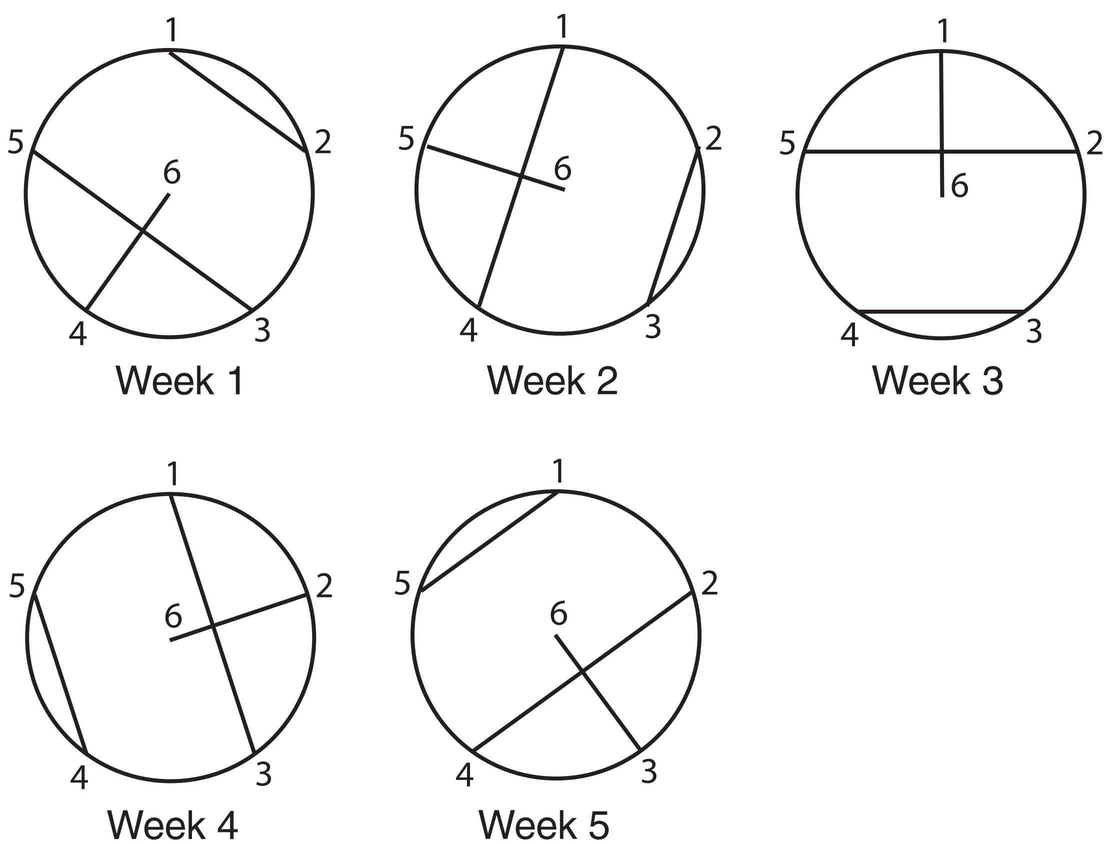


Week 4

Week 1: 1-2, 7-3, 6-4
Week 2: 2-3, 1-4, 7-5
Week 3: 3-4, 2-5, 1-6
Week 4: 4-5, 3-6, 2-7
Week 5: 5-6, 4-7, 3-1
Week 6: 6-7, 5-1, 4-2
Week 7: 7-1, 6-2, 5-3

COURSE NOTE 271

ROUND-ROBIN PAIRINGS
(for an even number of contestants)



Week 1: 1-2, 5-3, 4-6
Week 2: 2-3, 1-4, 5-6
Week 3: 3-4, 2-5, 1-6
Week 4: 4-5, 3-1, 2-6
Week 5: 5-1, 4-2, 3-6