Unit IV — Triangles

Appendix A – Properties of Real Numbers

Properties of the Real Numbers — This term refers to the postulates, or axioms, which we accepted without proof, in the study of Arithmetic. Following is a comprehensive list for your reference.

1. Properties of Relations:

- **Trichotomy** - For any real numbers a and b, only one of the following can be true: $a = b, a > b, a < b.$
- **Reflexivity for Equality** - For any real number a, $a = a.$
- **Symmetry for Equality** - For any real numbers a and b, if $a = b$, then $b = a.$
- **Transitivity for Equality** - For any real numbers $a, b,$ and $c,$ if $a = b,$ and $b = c,$ then $a = c.$
- **Substitution** - For any real numbers $a,$ and $b,$ if $a = b,$ then a can be substituted for b in any expression (or b can be substituted for a).
- **Transitivity for Inequality** - For any real numbers $a, b,$ and c, if $a > b,$ and $b > c,$ then $a > c.$ Likewise, if $a < b,$ and $b < c,$ then $a < c.$

2. Properties of Well-Defined Operations:

- **Existence** - For any real numbers a and $b, a + b, a - b,$ and $a \cdot b$ exist. Further, $a \div b$ exists, as long as $b = 0$.
- **Uniqueness** - For any real numbers a and b, $a + b, a - b,$ and $a \cdot b$ are unique. Further, $a \div b$ is unique, as long as $b = 0$.
- **Closure** - For any real numbers a and b, $a + b, a - b,$ and $a \cdot b$ are real numbers. Further, $a \div b$ is a real number, as long as $b = 0$.

3. Properties of Operations in General:

- **Commutativity of Addition** - For any real numbers a and $b, a + b = b + a.$
- **Commutativity of Multiplication** - For any real numbers a and $b, a \cdot b = a \cdot b.$
- **Associativity of Addition** - For any real numbers $a, b,$ and $c,$ $(a + b) + c = a + (b + c).$
- **Associativity of Multiplication** - For any real numbers $a, b,$ and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c).$
- **Distributivity of Multiplication over Addition** - For any real numbers $a, b,$ and $c,$ $a \cdot (b + c) = a \cdot b + a \cdot c.$
- **Distributivity of Multiplication over Subtraction** - For any real numbers $a, b,$ and $c, a \cdot (b - c) = a \cdot b - a \cdot c.$

4. Properties of Operations with Special Numbers:

- **Identity of Addition** - There exists a unique real number 0, such that, for every real number a, $a + 0 = a$.
- **Identity of Subtraction** - There exists a real number 0, such that, for every real number a, $a - 0 = a$.
- **Additive Inverse** - For every real number a, there exists a unique real number, $(-a)$, such that $a + (-a) = 0$.
- **Multiplication by Zero** - There exists a unique real number 0, such that, for every real number a, $a \cdot 0 = 0$.
- **Division by Zero** - Division by zero is meaningless and undefined. Therefore, it is not allowed.
- **Identity of Multiplication by 1** - There exists a unique real number 1, such that, for every real number a, $a \cdot 1 = a$.
- **Identity of Division by 1** - There exists a unique real number 1, such that, for every real number a, $a \div 1 = a$.
- **Multiplicative Inverse** - For every real number a, there exists a unique real number, $\frac{1}{a}$, such that, $a \cdot \frac{1}{a} = 1$.
- **Identity of a Power of 1** - There exists a unique real number 1, such that, for every real number a, $a^1 = a$.

5. Properties of Operations on Relations:

- **Addition** - For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$. Likewise, if $a > b$, then $a + c > b + c$, and, if $a < b$, then $a + c < b + c$.
- **Subtraction** - For any real numbers a, b, and c, if $a = b$, then $a - c = b - c$. Likewise, if $a > b$, then $a - c > b - c$, and, if $a < b$, then $a - c < b - c$.
- **Positive Multiplication** - For any real numbers a, b, and c, with $c > 0$, if $a = b$, then $a \cdot c = b \cdot c$. Likewise, if $a > b$, then $a \cdot c > b \cdot c$, and, if $a < b$, then $a \cdot c < b \cdot c$.
- **Positive Division** - For any real numbers a, b, and c, with $c > 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$. Likewise, if $a > b$, then $\frac{a}{c} > \frac{b}{c}$, and, if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.
- **Negative Multiplication**- For any real numbers a, b, and c, with $c < 0$, if $a > b$, then $a \cdot c < b \cdot c$, or, if $a < b$, then $a \cdot c > b \cdot c$.
- **Negative Division** - For any real numbers a, b, and c, with $c < 0$, if $a > b$, then $\frac{a}{c} < \frac{b}{c}$, or, if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

Note – With respect to relations in geometry, we can extend the properties in 1. above as follows:

Properties of Relations in Geometry:

1. Properties of Similarity

- **Reflexivity for Similarity** - For any geometric figure F, $F \sim F$.
- **Symmetry for Similarity** - For any two geometric figures, F and G, if $F \sim G$, then $G \sim F$.
- **Transitivity for Similarity** - For any three geometric figures, F, G, and H, if $F \sim G$, and $G \sim H$, then $F \sim H$.

2. Properties of Congruence

- **Reflexivity for Congruence** - For any geometric figure F, $F \cong F$.
- **Symmetry for Congruence** - For any two geometric figures, F and G, if $F \cong G$, then $G \cong F$.
- **Transitivity for Congruence** - For any three geometric figures, F, G, and H, if $F \cong G$, and $G \cong H$, then $F \cong H$.